

Exam 7



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Exam 7

Estimation of Policy Liabilities, Insurance Company Valuation, and ERM

May 2, 2018

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 59 point examination consists of 24 problem and essay questions.
2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid/tape.
 - Write your Candidate ID number and the examination number, 7, at the top of each answer sheet. For your Candidate ID number, four boxes are provided corresponding to one box for each digit in your Candidate ID number. If your Candidate ID number is fewer than 4 digits, begin in the first box and do not include leading zeroes. Your name, or any other identifying mark, must not appear.
 - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER.** Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as “Page 1 of 2” on the first sheet of paper and then “Page 2 of 2” on the second sheet of paper.
 - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
 - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.
3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

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4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.
5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.
6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.
7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.
8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a copy of the examination from the CAS Web Site.

All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.
9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 16, 2018.

END OF INSTRUCTIONS

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Exam 7, Spring 2018

1. (7.5 points)

ABC Insurance is a commercial insurance company with complex agent/broker distribution channels. ABC Insurance is exploring the acquisition of Target X to increase its distribution flexibility.

Target X is an innovative mobile app based insurer that allows small business owners to customize and purchase general liability and property policies from their cellphones without the aid of an agent/broker.

Target X has consistently achieved a 15% ROE and 6% growth rate. Assume the risk free rate is 4%, the equity market risk premium is 5%, and company β is 1.10.

Target X has provided ABC Insurance with its abbreviated balance sheet as of December 31, 2017:

Assets (\$000)		Liabilities (\$000)	
Cash	17,250	Unpaid Claims	5,000
Investments	3,250	Unpaid Claims Expenses	1,250
Intangible Assets	2,000	Unearned Premium Reserves	8,500
All Other Assets	1,000	All Other Liabilities	2,750

ABC Insurance has obtained Price-Book Value multiples for similar, recent transactions across the industry:

Acquisition	ROE	Growth Rate	Discount Rate	Price-Book Value
#1	15%	6%	9.5%	1.00
#2	10%	3%	9.5%	1.25
#3	15%	6%	13.9%	1.50
#4	15%	6%	9.5%	1.70
#5	15%	6%	2.3%	1.75
#6	5%	-2%	2.3%	1.90

Target X also provided ABC Insurance with the following data from its most recent unpaid claims analysis, which utilized an over-dispersed Poisson (ODP) bootstrap model with five parameters:

ODP Bootstrap Model Fitted Incremental Paid Claims (\$000) as of (months)			
Accident Year	12	24	36
2015	2,703	1,297	1,000
2016	3,547	1,703	
2017	4,250		

Unscaled Pearson Residuals as of (months)			
Accident Year	12	24	36
2015	0.91	-1.31	0
2016	-0.79	1.15	
2017	0		

Standardized Pearson Residuals as of (months)			
Accident Year	12	24	36
2015	2.12	-2.12	0
2016	-2.12	2.12	
2017	0		

Random Sample of Standardized Pearson Residuals as of (months)			
Accident Year	12	24	36
2015	2.12	2.12	-2.12
2016	-2.12	-2.12	
2017	-2.12		

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1. (continued)

a. (1 point)

A type of strategic risk posed by the acquisition above is project risk - the risk of entering an acquisition without contemplating integration costs, timelines, reserve deficiencies, etc. Describe two additional types of strategic risk specific to the acquisition above.

b. (1 point)

A primary cause of property & casualty insurer insolvency is deficient loss reserves. Describe two additional potential sources of insolvency specific to the acquisition above.

c. (1.25 points)

Create a reasonable range of firm values for Target X using the Price-Book Value transaction multiples.

d. (2 points)

Calculate the mean of the accident year 2017 incremental paid between 24 and 36 months for the given random sample of standardized Pearson residuals.

e. (1 point)

Calculate the variance of the accident year 2017 incremental paid between 24 and 36 months for the given random sample of standardized Pearson residuals.

f. (0.5 point)

Describe how to incorporate process variance in the accident year 2017 incremental paid between 24 and 36 months for the given random sample of standardized Pearson residuals.

g. (0.5 point)

Assume Target X's claims settlement rate will change after the acquisition. Briefly describe how this may impact the process risk and parameter risk of Target X's unpaid claims model.

h. (0.25 point)

Assume Target X's actuarial department will operate independently after the acquisition. Briefly describe the internal systemic risk correlation between Target X and ABC Insurance.

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Exam 7, Spring 2018

2. (2.75 points)

Given the following information for a workers compensation book written with a per-occurrence deductible of \$500,000:

Fitted Limited Severity		
Limit	36 months	Ultimate
Unlimited	\$5,000	\$9,000
\$500,000	\$4,500	\$6,600

	36 months	Ultimate
Observed Deductible Loss	\$1,000,000	-
Observed Excess Loss	\$275,000	-
Modeled Excess Age-to-Ultimate Factor	3.00	-
Insurance Charge Ratio	0.20	0.30

- Expected claims volume = 1,000 claims.

a. (2.25 points)

Assess the reasonability of the modeled excess IBNR compared to the loss ratio approach.

b. (0.5 point)

Given the following:

Volume Weighted Claim Count Age-to-Age Development Factors as of (months)				
12-24	24-36	36-48	48-60	60-72
1.50	1.25	1.15	1.05	1.00

Describe an issue with using limited severity data at 36 months.

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3. (2 points)

Given the following information as of December 31, 2017:

AY	Earned Premium (\$000)	Cumulative Paid Loss (\$000) as of (months)			
		12	24	36	48
2014	8,000	2,500	3,335	3,942	4,021
2015	8,320	2,100	2,705	3,335	
2016	8,650	3,000	4,113		
2017	9,000	3,500			

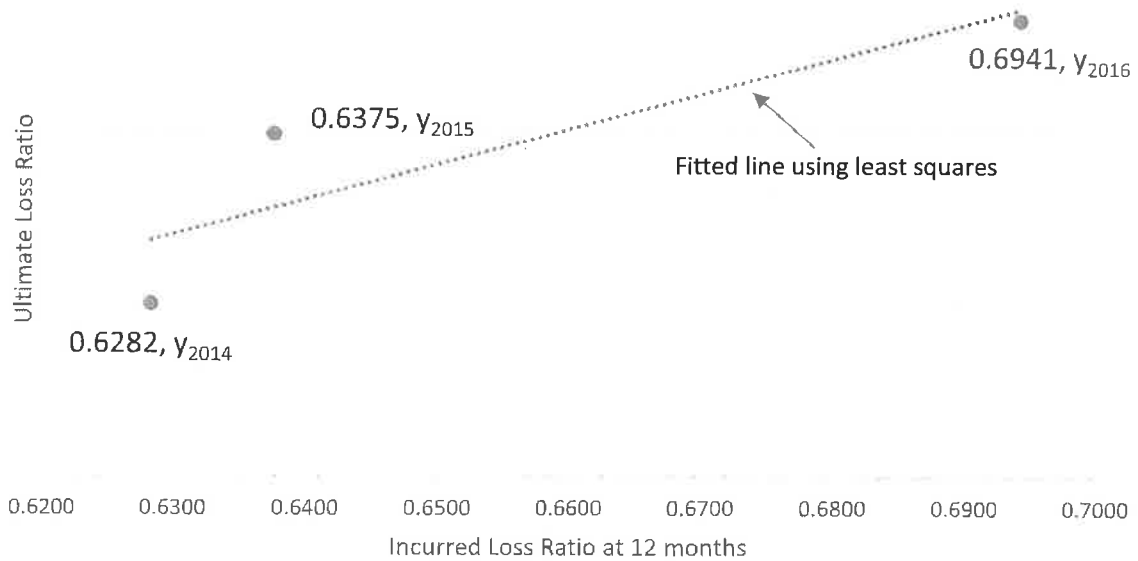
- Assume there is no further development after 48 months.
- $t_i = \sqrt{p_i}$
- $E[\alpha_2^2 (U_2)] = 2,000$

Calculate the mean squared error for both the individual loss ratio method and the collective loss ratio method, and determine which is preferable for estimating R_{2015} .

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4. (2 points)

Given the following information:



- The equation for the line in the graph above is $y = 0.1126 + bx$.
- $\bar{xy} = 0.5979$
- Cumulative incurred loss for accident year 2017 at 12 months is \$6,000,000.
- Earned premium for accident year 2017 is \$8,000,000.
- Assume there is no further development after 24 months.

Calculate the ultimate loss for accident year 2017 using the least squares method.

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5. (2.25 points)

Given the following information about accident year 2017 as of December 31, 2017:

- Accident year 2017 paid loss = \$850,000.
- 2017 earned premium = \$4,000,000.
- Initial expected loss ratio = 67.5%.
- 12-24 month incremental paid link ratio = 1.60.
- 12-ultimate cumulative paid LDF = 3.00.

a. (1.5 points)

Determine the accident year 2017 incremental paid loss in 2018 that would result in the Benktander ultimate loss estimate being \$100,000 less than the Bornhuetter-Ferguson ultimate loss estimate for accident year 2017 as of December 31, 2018. Assume all development factors are unchanged.

b. (0.25 point)

Briefly describe when the Benktander ultimate loss estimate would be greater than the Bornhuetter-Ferguson ultimate loss estimate as of December 31, 2018.

c. (0.5 point)

Explain why it may not be appropriate to use the Bornhuetter-Ferguson method when losses develop downward.

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Exam 7, Spring 2018

6. (3 points)

Given the following information for an insurer's books of business as of December 31, 2017:

Accident Year	Onlevel Premium (\$000)	Cumulative Paid Loss (\$000)	Estimated Reserves (\$000)
2014	1,000	275	400.00
2015	1,200	306	553.85
2016	1,500	344	818.18
2017	1,700	220	1,133.33

- The estimated reserves for all accident years are calculated using the Cape Cod method.
- The expected loss payment pattern is approximated by the following loglogistic function where G is the cumulative proportion of ultimate losses paid and x represents the average age of paid losses in months: $G(x) = \frac{x}{x + \theta}$.

a. (1.5 points)

Calculate the expected loss ratio used in the Cape Cod method.

b. (0.75 point)

Evaluate the appropriateness of using the Cape Cod method for this book of business.

c. (0.75 point)

Briefly describe the two types of variance associated with a statistical model for loss reserving. Identify an approach to reduce one of the types of variance.

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7. (2.5 points)

Given the following information:

	Cumulative Reported Loss (\$000) as of (months)				
Accident Year	12	24	36	48	60
2012	1,300	2,350	3,500	3,650	3,750
2013	1,450	3,000	5,050	5,850	
2014	1,600	2,500	3,700		
2015	1,750	3,050			
2016	1,950				

	Observed Loss Development Factors as of (months)			
Accident Year	12-24	24-36	36-48	48-60
2012	1.808	1.489	1.043	1.027
2013	2.069	1.683	1.158	
2014	1.563	1.480		
2015	1.743			

- There is no development beyond 60 months.

a. (0.5 point)

Calculate the ultimate loss for AY 2014 under the assumption that the variance of the next incremental loss is a constant for each development age.

b. (0.5 point)

Calculate the ultimate loss for AY 2014 under the assumption that the variance of the next incremental loss is proportional to the losses reported to date.

c. (0.5 point)

Calculate the ultimate loss for AY 2014 under the assumption that the variance of the next incremental loss is proportional to the square of the losses reported to date.

d. (1 point)

Fully describe a test to determine which of the three variance assumptions in parts a., b., and c. above is most reasonable for the data in this triangle. Do not perform any calculations.

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8. (2.75 points)

Given the following information:

Observed Age-to-Age Loss Development Factors as of (years)				
AY	1-2	2-3	3-4	4-5
1	1.700	1.350	1.100	1.050
2	2.500	1.550	1.080	
3	2.000	1.400		
4	1.800			

- There is no development after the 5th period.
- Probability of a standard normal variate lying in the interval $(-0.67, 0.67)$ is 50%.

a. (2 points)

Evaluate whether subsequent columns of development factors exhibit correlation using Spearman's rank at a 50% confidence threshold.

b. (0.5 point)

Briefly describe two reasons why it is more appropriate to test the triangle as a whole rather than correlation between pairs of columns.

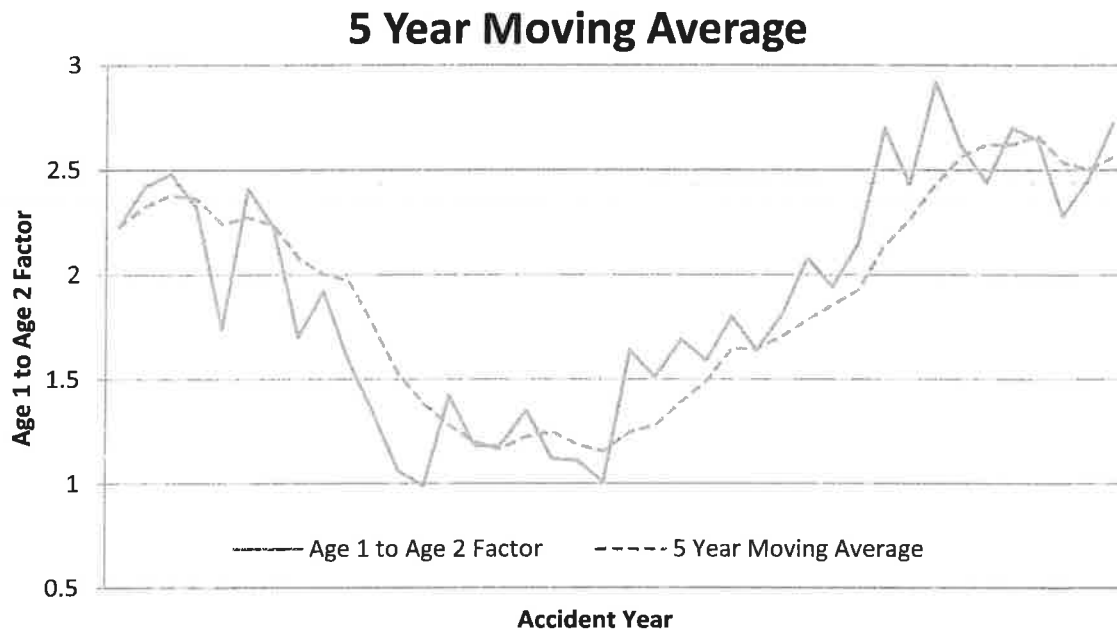
c. (0.25 point)

Identify which assumption of the chain ladder method is being evaluated in part a. above.

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9. (1.25 points)

Given the following information:



- The all-year weighted average is used to calculate the age 1 to age 2 factor.
- An actuary believes that the all-year weighted average is inappropriate.

a. (0.75 point)

Support the actuary's claim and provide two methods that could improve the estimated age-to-age factor considering the chart above.

b. (0.5 point)

Describe an alternative method of testing the stability of the age-to-age factors based on Venter Factors.

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10. (2.5 points)

Given the following information as of December 31, 2017:

Accident Year	Incremental Paid Loss (\$000) as of (months)	
	12	24
2013	600	1,300
2014	650	1,400
2015	550	1,400
2016	700	1,450
2017	700	

- Exposures are stable over the years.
- There are no calendar or accident date trends in the data.
- A simple linear regression $y = a + bx$ of the incremental loss at age 24 over the cumulative loss at age 12 was performed. The resulting regression coefficients are:
 - $a = 1,075$
 - $b = 0.5$
- For a simple linear regression $y = a + bx$, the standard error for the slope (b) and intercept (a) are calculated as:

$$s_b = \sqrt{\frac{\sum_i \epsilon_i^2}{(n-2) \sum_i (x_i - \bar{x})^2}}$$

$$s_a = s_b \sqrt{\frac{1}{n} \sum_i x_i^2}$$

where ϵ_i are the error terms.

a. (2 points)

Test for significance of the 12-24 loss development factor and state the chain ladder model assumption that this test is used to validate.

b. (0.5 point)

Discuss how the estimated future loss emergence for a year with abnormally high losses to date would differ under the chain ladder versus additive model.

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11. (1.75 points)

An actuary is using a GLM bootstrap model to estimate the reserves for a line of business. The model residuals by accident year and development period are:

Accident Year	Model Residuals as of (months)				
	12	24	36	48	60
2013	100	-100	-50	0	50
2014	100	-100	-20	10	
2015	-80	80	0		
2016	45	-70			
2017	0				

a. (0.5 point)

Briefly describe heteroscedasticity and identify an issue it may cause when using a GLM bootstrap model.

b. (0.75 point)

Evaluate whether development period heteroscedasticity exists.

c. (0.5 point)

Describe a method to adjust for heteroscedasticity.

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12. (2 points)

An insurer is using the following information without adjustment to build a stochastic loss reserve model for its property exposures in a coastal area:

Accident Year	Number of Hurricanes	Number of All Other Peril Events	Losses from Hurricanes (\$000)	Losses from All Other Perils (\$000)
2012	0	4,750	0	54,625
2013	1	5,150	250,000	60,255
2014	0	5,000	0	60,000
2015	0	5,300	0	65,190
2016	0	4,900	0	61,250
Total	1	25,100	250,000	301,320

- The data is valued as of June 30, 2017.
- The underlying exposures for the book of business are not expected to change.
- The resulting model produced an R^2 value greater than 99%.
- The insurer will perform a retrospective test of the model using ten years of data valued as of June 30, 2027.
- Assume no data errors.

a. (1 point)

Identify and briefly describe two sources of internal systemic risk applicable to this model.

b. (1 point)

Draw a histogram assuming no hurricane losses in the next ten years and briefly explain your selection of the shape.

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13. (2.75 points)

An insurer uses the following information in an over-dispersed Poisson (ODP) bootstrap process by building a generalized linear model (GLM) based on incremental paid loss.

Cumulative Reported Loss (\$000) as of (months)			
Accident Year	12	24	36
2015	180	230	235
2016	200	180	
2017	225		

Open Claims as of (months)			
Accident Year	12	24	36
2015	3	1	0
2016	5	3	
2017	6		

Case reserves are set based on the maturity of the claim:

- 12 months = \$15,000 per claim
- 24 months = \$10,000 per claim
- 36 months = \$0 per claim.

a. (0.5 point)

Briefly describe two practical limitations of a log-link GLM bootstrap modeling framework.

b. (1.25 points)

Construct a log-link triangle that can be used to solve for the GLM parameters.

c. (1 point)

Calculate the fitted incremental paid results of the GLM model using the following parameters:

- $\alpha_1 = 5.04$
- $\alpha_2 = 4.66$
- $\alpha_3 = 4.91$
- $\beta_2 = -0.86$
- $\beta_3 = -1.47$

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14. (1.25 points)

a. (0.5 point)

A claims person, faced with insufficient and possibly conflicting information about a potentially serious claim, may tend to reserve to “expectation,” which is most likely interpreted by the claims person as the mode of the probability distribution.

Briefly describe two ways this practice can make reinsurance reserving difficult.

b. (0.75 point)

Briefly describe three reasons heterogeneous exposures assumed by a reinsurance company can make reinsurance reserving difficult.

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15. (2.25 points)

For each of the scenarios in parts a., b., and c. below, briefly describe:

- i. The likely impact on the loss data,
- ii. The adjustment that should be made by incorporating expert opinion, and
- iii. The impact on IBNR estimates if no adjustments are made.

a. (0.75 point)

Claims staffing level has increased over the last year relative to the previous four years.

b. (0.75 point)

The company's property portfolio has grown rapidly in catastrophe-prone areas.

c. (0.75 point)

A large claim reserved at \$10M ultimately settles favorably for \$1M.

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16. (3 points)

Given the following information:

Accident Year	Rate-Level Adjusted Premium (\$000)	Aggregate Reported Loss at Dec. 31, 2016 (\$000)	Aggregate Loss Report Lag at Dec. 31, 2016	Aggregate Reported Loss at Jun. 30, 2017 (\$000)	Aggregate Loss Report Lag at Jun. 30, 2017
2014	6,000	3,000	0.70	3,500	0.75
2015	7,000	2,000	0.50	2,200	0.55
2016	10,000	4,000	0.30	4,800	0.35
Total	23,000	9,000	N/A	10,500	N/A

- The selected credibility factor is 0.50.

a. (2.5 points)

Test the total expected claims emergence from December 31, 2016 to June 30, 2017 against the total actual claims emergence using a credibility-weighted combination of the Stanard-Bühlmann and chain ladder methods.

b. (0.5 point)

Provide two reasons which would explain the difference calculated in part a. above.

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17. (2.25 points)

Given the following information for a retrospectively rated book of business:

Policy Effective Year	Ultimate Loss	Losses Reported at Prior Retro Adjustment	Premiums Booked from Prior Adjustment	Premiums Booked as of December 31, 2017
2014	\$226,500	\$202,000	\$335,000	\$337,000
2015	\$225,000	\$180,500	\$330,000	\$335,000
2016	\$200,000	0	0	\$210,000
2017	\$80,000	0	0	\$215,000

Retro Adjustment Period	Selected PDLR Ratio	Selected CPDLR Ratio	Percentage of Ultimate Loss Reported
First	1.75	???	78.5%
Second	0.70	0.59	10.0%
Third	0.55	0.49	7.0%
Fourth	0.45	0.40	4.0%
Subsequent	0.00	0.00	0.5%

- Retro premium is booked 24 months after policy effective dates.

a. (1.25 points)

Calculate the premium asset as of December 31, 2017.

b. (1 point)

Describe two reasons why the selected PDLR ratios decrease as the retro adjustment period increases.

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18. (3.5 points)

Given the following financial information for a P&C insurance company as of December 31, 2017:

- Firm's projected ROE = 12.0%.
- Projected dividend payout rate = 50.0%.
- Yield on long-term T-bonds = 3.0%.
- Liquidity premium for long-term T-bonds = 1.0%.
- Expected equity risk premium = 6.0%.
- Insurer's equity beta for the Company = 1.5.
- Insurer's book value as of December 31, 2017 = \$100M.

a. (1 point)

Calculate the company's Price to Book Value (P-BV) multiple, assuming projections continue into perpetuity.

b. (0.5 point)

Calculate the company's Price to Book Value (P-BV) multiple, assuming excess returns, if any, decline to the cost of capital after 3 years.

c. (0.5 point)

Identify and briefly discuss which Price to Book Value multiple would serve as a more reasonable indicator in the valuation of the company considering parts a. and b. above.

d. (0.5 point)

Describe one reason why transaction multiples based on initial public offerings or mergers and acquisitions of peer firms may be a better reflection in the valuation of the company than the approaches used in parts a. and b. above.

e. (1 point)

Describe two reasons why transaction multiples based on initial public offerings or mergers and acquisitions of peer firms may not be better reflections in the valuation of a company than in the approaches used in parts a. and b. above.

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19. (2.5 points)

A company's equity is valued at \$276,716,000 based on the following assumptions:

	2018	2019	2020
Net Income (\$000)	22,000	28,000	34,000
Starting GAAP Equity (\$000)	189,200	190,300	191,700

- The risk free interest rate = 4.0%.
- The equity market risk premium = 5.0%.
- The company's beta = 0.8.
- After 2020, the company's abnormal earnings decrease linearly until reaching zero in 2031.

Financial ratios for several peer companies are summarized below:

	Price/Earnings	Price/Sales	Price/Cash Flow	Price/Book
Peer 1	12.5	1.0	8.0	1.29
Peer 2	13.4	0.9	7.7	1.59
Peer 3	12.5	2.1	14.1	1.37
Peer 4	12.9	4.5	12.8	1.76

a. (1.5 points)

Calculate the change in the estimated equity value if the abnormal earnings are assumed to decrease linearly to zero in 2023 instead of in 2031.

b. (1 point)

Justify whether 2 years or 10 years of non-zero abnormal earnings after 2020 produces a more reasonable answer.

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20. (1.5 points)

An insurance company does not have a reinsurance program in place. Based on the latest Enterprise Risk Management review, management wants to reduce its insurance hazard risk by purchasing reinsurance and is reviewing two reinsurance programs.

An actuary estimates the following information based on the modeled probability distribution of the underwriting results:

	Current	Option 1	Option 2
Direct Premium (\$000)	1,000	1,000	1,000
Direct Loss (\$000)	600	600	600
Net Expenses (\$000)	200	200	200
Expected Ceded Premium (\$000)	0	220	330

Expected Net Combined Ratio	80.0%	83.0%	86.0%
1-in-100 TV@R Net Loss (\$000)	2,000	1,500	1,100

- The company uses 1-in-100 TV@R to set its required capital.

Calculate the insurance company's marginal ROEs for reinsurance Option 1 and Option 2.

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21. (1.75 points)

A company writes two lines of business: personal auto and personal property. An actuary simulated 1,000 loss scenarios of a company's personal property line of business using an Internal Risk Model. Given the following information:

- 1-in-100 TV@R of personal auto = \$4.5 million.
- 1-in-100 WTV@R of personal auto = \$5 million.
- The company has a total capital of \$10 million.
- All scenarios have an equal chance of occurring.
- The actuary calculated the transformed probability function ($F^*(x)$) using Wang transform.
- The results of the 13 worst scenarios sorted in descending order for personal property:

Simulation #	Loss	$F^*(x)$
709	6,933,000	1.000
330	5,606,000	0.995
792	4,867,000	0.991
424	4,121,000	0.987
114	3,022,000	0.984
185	2,496,000	0.981
201	1,800,000	0.978
977	1,293,000	0.975
644	1,022,000	0.972
663	836,000	0.969
109	423,000	0.966
514	383,000	0.963
577	200,000	0.961

a. (0.75 point)

Calculate the allocated capital for each of the lines of business using the proportional allocation method with 1-in-100 TV@R.

b. (1 point)

Calculate the allocated capital for each of the lines of business using the proportional allocation method with 1-in-100 WTV@R.

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22. (3 points)

An insurance company needs to strengthen its capital position in order to maintain its financial strength rating. The company currently has no reinsurance and is considering the following two strategies:

Strategy 1: Issue an \$80 million surplus note

- Coupon rate on surplus note = 8%.
- Bond yield = 5%.
- Coupons are paid out annually.
- The surplus note cannot be repaid for a minimum of 5 years.

Strategy 2: Purchase quota-share reinsurance

- Gross earned premium = \$200 million.
- Economic margin on contract = 15% of ceded premium.
- The insurance company will cede 10% of its earned premium in the first year.
- It is projected that the company's profits will strengthen its surplus over time and the cession will be reduced to 8% in the second year, 3% in the third year, and eliminated in the fourth year.
- The projected annual gross earned premium is stable over the next five years.

a. (0.75 point)

Calculate the cost of each strategy in the first year and determine which strategy is preferred on a one-year basis.

b. (1.25 points)

Calculate the cost of each strategy over five years, using the bond yield as a discount rate, and determine which strategy is preferred on a five-year basis.

c. (1 point)

Identify the three paradigms for measuring the value in a reinsurance structure and identify which of the paradigms was used to evaluate the quota share reinsurance structure in Strategy 2 above.

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23. (2 points)

A large insurance company has developed an Enterprise Risk Management (ERM) model which has the following characteristics:

- i. The ERM model will be maintained by internal employees across various functions (including actuarial, underwriting and planning) who will work on the model in addition to their regular duties.
- ii. The insurance company is exposed to a significant amount of earthquake risk. To quantify its earthquake risk, the company uses a single earthquake model from a third-party vendor.
- iii. The ERM model incorporates correlation among the insurance company's different lines of business using parameters developed individually by each line of business leader.
- iv. The insurance company reviews the simulation results from the ERM model and sets its capital requirements based solely on the amount required to achieve a sufficiently small probability of default.

Describe one weakness of each characteristic above and propose a possible improvement to address each weakness.

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24. (1 point)

An insurance company follows a “maintain market share” strategy during a soft market.

a. (0.5 point)

Briefly describe the strategy and provide a negative implication in a soft market.

b. (0.5 point)

Briefly describe two features of a more effective underwriting cycle management strategy.

END OF EXAMINATION

Exam 7

Estimation of Policy Liabilities, Insurance Company Valuation, and Enterprise Risk Management

POINT VALUE OF QUESTIONS

QUESTION	VALUE OF QUESTON	SUB-PART OF QUESTION							
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
1	7.50	1.00	1.00	1.25	2.00	1.00	0.50	0.50	0.25
2	2.75	2.25	0.50						
3	2.00	2.00							
4	2.00	2.00							
5	2.25	1.50	0.25	0.50					
6	3.00	1.50	0.75	0.75					
7	2.50	0.50	0.50	0.50	1.00				
8	2.75	2.00	0.50	0.25					
9	1.25	0.75	0.50						
10	2.50	2.00	0.50						
11	1.75	0.50	0.75	0.50					
12	2.00	1.00	1.00						
13	2.75	0.50	1.25	1.00					
14	1.25	0.50	0.75						
15	2.25	0.75	0.75	0.75					
16	3.00	2.50	0.50						
17	2.25	1.25	1.00						
18	3.50	1.00	0.50	0.50	0.50	1.00			
19	2.50	1.50	1.00						
20	1.50	1.50							
21	1.75	0.75	1.00						
22	3.00	0.75	1.25	1.00					
23	2.00	2.00							
24	1.00	0.50	0.50						
TOTAL	59.00								

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

GENERAL COMMENTS:

- Candidates should note that the instructions to the exam explicitly say to show all work; graders expect to see enough support on the candidate's answer sheet to follow the calculations performed. While the graders made every attempt to follow calculations that were not well-documented, lack of documentation may result in the deduction of points where the calculations cannot be followed or are not sufficiently supported.
- Candidates should justify all selections when prompted to do so. For example, if the candidate selects an all year average and the question prompts a justification of all selections, a brief explanation should be provided for the reasoning behind this selection. Candidates should note that a restatement of a numerical selection in words is not a justification.
- Incorrect responses in one part of a question did not preclude candidates from receiving credit for correct work on subsequent parts of the question that depended upon that response.
- Candidates should try to be cognizant of the way an exam question is worded. They must look for key words such as "briefly" or "fully" within the problem. We refer candidates to the Future Fellows article from December 2009 entitled "The Importance of Adverbs" for additional information on this topic.
- Some candidates provided lengthy responses to a "briefly describe" question, which does not provide extra credit and only takes up additional time during the exam.
- Candidates should note that the sample answers provided in the examiner's report are not an exhaustive representation of all responses given credit during grading, but rather the most common correct responses.
- In cases where a given number of items were requested (e.g., "three reasons" or "two scenarios"), the examiner's report often provides more sample answers than the requested number. The additional responses are provided for educational value, and would not have resulted in any additional credit for candidates who provided more than the requested number of responses. Candidates are reminded that, per the instructions to the exam, when a specific number of items is requested, only the items adding up to that number will be graded (i.e., if two items are requested and three are provided, only the first two are graded).
- It should be noted that all exam questions have been written and graded based on information included in materials that have been directly referenced in the official syllabus, which is located on the CAS website. The CAS takes no responsibility for the content of supplementary study materials and/or manuals produced by outside corporations and/or individuals which are not directly referenced in the official syllabus.

EXAM STATISTICS:

- Number of Candidates: 897
- Available Points: 59.0
- Passing Score: 42.25
- Number of Passing Candidates: 343
- Raw Pass Ratio: 38.2%
- Effective Pass Ratio: 40.3%

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 1	
TOTAL POINT VALUE: 7.5	LEARNING OBJECTIVE(S): A5, A7, B3, C7
SAMPLE ANSWERS	
Part a: 1 point	
<u>Sample responses:</u>	
<ul style="list-style-type: none"> • Technology [or IT]: if another company patents the technology of buying insurance from a cell phone, then others will not be allowed to utilize this technology. • Reputation [or Brand] risk: collapse or erosion of reputation. Since it is a new distribution, many unforeseen problems would arise and damage ABC reputation. • Competition [or Competitor] risk: other competitors may enter the market using similar apps to offer lower prices. • Customers [or Client] and intermediary: the customers of ABC are used to having intermediaries such as agents or brokers to service them. With the acquisition, the core focus might be divided and agents/brokers unhappy as the new distribution bypasses them. • Industry risk: if the market is currently a soft market in the underwriting cycle, increasing market share will worsen the company performance. Industry risk is the risk within the industry that affects all companies within the industry. • Stagnation: currently, the firm has a 6% growth rate, but this might decrease to 0 in the next years, resulting to a stagnation. 	
Part b: 1 point	
<u>Sample responses:</u>	
<ul style="list-style-type: none"> • Pricing inadequate → new line, so not much data → data limited → error in pricing → inadequate price → insolvency → Q.E.D. • Catastrophe Event: A significant catastrophe can easily generate large claims to an insurer and severely reduce its solvency. • Rapid Growth [or too fast growth]. Growing too fast can lead to deficiency, as won't have time to learn about your book. So this new book may have different development, different trends, so if grow too fast want to take those into account & will be mispriced for the losses coming in. • Reinsurance failure: not being able to recover ceded losses from the reinsurer may lead to insolvency in times of large catastrophes. • Underwriting: if ABC doesn't properly price target X's product (which essentially is new LOB for them i.e. inexperienced) may not have good uw results. • External Fraud: Since Target X is an app based insurer, ABC may lack expertise to detect claims fraud for claims filed on apps. • Overstated Assets: Target X will have provided overstated assets to ABC, which would skew ABC's perception of X thinking it is a good acquisition, even though in reality it is risky. • Reckless Management: Target X could have reckless management that may be making decisions that are not appropriate for firm's interests. • Insolvency of subsidiary: Groups can be rendered insolvent if one of their daughter companies goes insolvent. 	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part c: 1.25 points

Sample 1

Discount Rate $K = 0.04 + 1.1 \times 0.05 = 0.095$

Comparable transactions are #1 & #4

Equity for target X = Assets – Liabilities

$$17,250 + 3,250 + 1,000 - 5,000 - 2,750 - 8,500 - 1,250 = 4,000$$

$$\text{Range of firm values} = [4,000 \times 1, 4,000 \times 1.7] = [4,000, 6,800]$$

Sample 2

Discount Rate $K = 4\% + 1.1 \times 5\% = 9.5\%$

Comparable transactions are #1 & #4

Equity for target X = Assets – Liabilities, (excluding Intangible Assets)

$$17,250 + 3,250 + 1,000 - (5,000 + 2,750 + 8,500 + 1,250) = 4,000$$

$$\text{Range of firm values} = [4,000 \times 1, 4,000 \times 1.7] = [4,000, 6,800]$$

Part d: 2 points

Sample 1

Accident	12	24	36
<u>Year</u>	<u>Months</u>	<u>Months</u>	<u>Months</u>
2015	$2,813 = 2.12 \times \sqrt{2,703} + 2,703$	1,373	933
2016	3,421	1,616	
2017	4,112		

$$12 - 24 \text{ LDF} = \frac{2,813 + 1,373 + 3,421 + 1,616}{2,813 + 3,421} = 1.479$$

$$24 - 36 \text{ LDF} = \frac{2,813 + 1,373 + 933}{2,813 + 1,373} = 1.223$$

$$\text{Mean} = 4,112 \times 1.479 \times (1.223 - 1) = 1,356$$

Sample 2

Accident	12	24	36
<u>Year</u>	<u>Months</u>	<u>Months</u>	<u>Months</u>
2015	$2,813 = 2.12 \times \sqrt{2,703} + 2,703$	1,373	933
2016	3,421	1,616	
2017	4,112		

	12-24	24-36
LDF	1.48	1.22
% reported	0.553	0.818

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

$$\text{Mean} = \frac{4,112}{0.553} \times (1 - 0.818) = 1,356$$

Part e: 1 point

Sample 1

$$\text{Var} = \phi \times \text{Mean}$$

$$\frac{0.91^2 + (-1.31)^2 + 0^2 + (-0.79)^2 + 1.15^2 + 0^2}{6 - 5} = 4.491$$

$$\text{Var} = 4.991 \times \$1,356 = \$6,088$$

Sample 2

$$\text{Var} = \phi \times \text{Mean}$$

$$\phi = \frac{2.12^2 + (-2.12)^2 + 0^2 + (-2.12)^2 + 2.12^2 + 0^2}{6} = 2.996$$

$$\text{Var} = 2.996 \times \$1,356 = \$4,062$$

Part f: 0.5 point

Sample 1

We have calculated the mean in part (d) and variance in part (e). We can then match the moments to a gamma distribution and use a random number from the Gamma distribution to simulate an incremental paid loss value. This includes process variance.

Sample 2

Sample residuals from distribution of Gamma ($m^{\text{iter}}, \phi m^{\text{iter}}$) \rightarrow Gamma(1356, 6088).

Sample 3

Simulate from gamma distribution $q_{2017,24-36} \rightarrow$ Gamma(mean=1356, var=6088).

Part g: 0.5 point

Sample responses for Parameter Risk

- Parameter Risk will increase as parameter estimate based on historical data does not reflect the current claims settlement pattern.

Sample responses for Process Risk

- Process risk is the pure effect of randomness associated with the insurance process. Since the change in settlement rate is not a change in the insurance process it does not impact this risk.
- Process risk will decrease if claim settlement rate increases as reserve will decrease, thus process variance will decrease.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part h: 0.25 point
<u>Sample responses:</u> <ul style="list-style-type: none">• Correlation will be low since they are eliminating the same actuary effect with different departments.• Data errors – if both companies rely on the same source data there could still be correlation due to data errors.
EXAMINER'S REPORT
Candidates were expected to tie in knowledge from multiple readings across all three Learning Objectives, including showing a basic understanding of strategic risks, insolvencies, standardized Pearson residuals, process variance, and internal systemic risks.
Part a
Candidates were expected to know types of strategic risks and be able to provide details related to the specific insurance situation of a merger and acquisition. Common mistakes included: <ul style="list-style-type: none">• Citing plausible, real world strategic risks that were not able to be tied back to items provided in the stem of the question (e.g. catastrophe risk, estimation risk, parameter risk).• Identifying a subclass of the Project Risk given in the question. Discussions about IT integration, M&A problems, and cultural integration are all a subset within the process risk. Candidates were expected to identify uniquely different risks from process risks.• Naming a specific strategy risk while providing the definition for a different risk (e.g. listing Technology risk but then describing a competitor risk).
Part b
Candidates were expected to correctly identify and describe some of the primary reasons for impairments/insolvencies as mentioned in Brehm and provide details on the relevancy to the specific case. Common mistakes included: <ul style="list-style-type: none">• Listing only one or no reason.• Listing strategic risks instead of primary risks of impairment/insolvency.• Providing reasons that did not describe how it related to impairment (e.g. rapid growth: rapid growth is a leading cause of impairment).• Listing risks that could happen, but were not identified as primary risks or substantially similar (e.g. Unearned Premium reserve problems, asset risk (but not overstated assets), latent claims, investment risk, M&A risk, parameter risk, estimation risk).• Duplicating risks, or using the same risk with different phrasing (e.g. Cat risk and Event risk; internal fraud and external fraud).
Part c
Candidates were expected to use the information provided to determine an appropriate range of values for a target acquisition by evaluating Target X against other recent transactions with the

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

same ROE, growth rate, and discount rate. Candidates were expected to calculate the book value of Target X and use the price to book value ratios to determine a reasonable range for the firm value using those pieces of information.

Common mistakes included:

- Not calculating the discount rate to use to evaluate acquisitions.
- Including intangible assets in the book value calculation.
- Not providing a range of values.

Part d

Candidates were expected to recognize the need to calculate the random sample incremental triangle using the random sample of residuals provided, and then calculate LDFs.

Common mistakes included:

- Not calculating the random sample triangle, instead using the given fitted claims to calculate the LDFs.
- Calculating only LDFs or incremental paid amounts using the given fitted triangle.
- Incorrectly calculating the random sample triangle using the standardized or unscaled residuals.
- Calculating the incremental paid for the wrong accident year or development period.

Part e

Candidates were expected to know the relationship $\text{Var} = \phi \times \text{Mean}$ for the ODP distribution, and the formula to calculate the scale factor, ϕ . There are two acceptable formulas provided for calculating ϕ in the syllabus; either was accepted:

$$\phi = \frac{\sum(\text{unscaled Pearson residuals})^2}{N - p}$$

$$\phi = \frac{\sum(\text{standardized Pearson residuals})^2}{N}$$

Common mistakes included:

- Mixing up the two formulas for ϕ .
- Trying to recalculate the unscaled Pearson residuals to solve for ϕ , rather than using the information provided. This approach doesn't work because the information needed to calculate the residuals is not provided in the problem.
- Using the wrong values of N (data points) or p (parameters) in the formula.

Part f

Candidates were expected to give a specific description of how to incorporate process variance by simulating incremental paid amounts from a distribution fit to the mean and variance calculated in parts (d) and (e).

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Common mistakes included:

- Providing an answer that was too general, rather than for the specific accident year 2017 24-36 month increment asked in the question.
- Insufficiently explaining how the distribution would be utilized (e.g. simply saying “use a gamma(1356, 6084)”).

Part g

Candidates were expected to recognize that a change in settlement rate would impact parameter risk but not necessarily process risk.

Common mistakes included:

- Mixing up process and parameter risk.
- Stating that process risk will increase or decrease without providing support.

Part h

Candidates were expected to recognize that by operating independently the two departments would likely have a reduced correlation of internal systemic risk.

Common mistakes included:

- Listing external or macro risks as reasons for increased internal systemic risk.
- Listing various types of internal systemic risks without addressing the situation in question.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 2	
TOTAL POINT VALUE: 2.75	LEARNING OBJECTIVE(S): A4
SAMPLE ANSWERS	
Part a: 2.25 points	
<i>Sample responses are separated into the three main pieces of the question:</i>	
Modeled XS IBNR:	
<u>Sample 1</u>	
XS IBNR = Reported XS Loss × Modeled XS LDF – Reported XS LDF = 275K × 3 – 275K = 550K	
<u>Sample 2</u>	
XS IBNR = Reported XS Loss × Derived XS LDF – Reported XS LDF = 275K × 4.806 – 275K = 1,045K	
$XSLDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{1 - R_{ult}^{500}}{1 - R_{36}^{500}} = 1.80 \times (1 - 0.733) / (1 - 0.900) = 4.806$	
$R_{ult}^{500} = 6,600 / 9,000 = 0.733$	
$R_{36}^{500} = 4,500 / 5,000 = 0.900$	
$LDF_{36-ult}^{unlim} = 9,000 / 5,000 = 1.80$	
IBNR from LR Approach:	
<u>Sample 1</u>	
$XSLDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{1 - R_{ult}^{500}}{1 - R_{36}^{500}} = 1.80 \times (1 - 0.733) / (1 - 0.900) = 4.806$	
$R_{ult}^{500} = 6,600 / 9,000 = 0.733$	
$R_{36}^{500} = 4,500 / 5,000 = 0.900$	
$LDF_{36-ult}^{unlim} = 9,000 / 5,000 = 1.80$	
XS Ult Loss = XS Sev × Counts = (9,000 – 6,600) × 1,000 = 2.4M	
XS IBNR = 2.4M × (1 – 1 / 4.806) = 1,900,624	
<u>Sample 2</u>	
Ult XS = (Ult Total Sev – Ult Limited Sev) × Counts = (9,000 – 6,600) × 1,000 = 2.4M	
XS@36 mo = (Total Sev@36 mo – Limited Sev@36 mo) × Counts = (5,000 – 4,500) × 1,000 = 0.5M	
XS IBNR = 2.4M – 0.5M = 1.9M	
<u>Sample 3</u>	
Ult XS = (Ult Total Sev – Ult Limited Sev) × Counts = (9,000 – 6,600) × 1,000 = 2.4M	
XS IBNR = Ult XS – Rept XS = 2.4M – 275K = 2,125K	
<u>Sample 4</u>	
Ult XS = (Ult Total Sev – Ult Limited Sev) × Counts = (9,000 – 6,600) × 1,000 = 2.4M	
XS IBNR = Ult XS × (1 – 1 / Modeled XS LDF) = 2.4M × (1 – 1/3.00) = 1.6M	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 5

LR approach = $P \cdot E \cdot \chi$ - Rept XS

Ult XS = Ult Total Sev \times Counts \times Ins Charge@Ult = $9,000 \times 1,000 \times 0.30 = 2.7M$

Rept XS = Total Report Loss \times Ins Charge@36 mo = $(1M + 275K) \times 0.20 = 255K$

XS IBNR = $2.7M - 255K = 2,445K$

Sample 6

LR approach = $P \cdot E \cdot \chi$ - Rept XS

Derived Unlim LDF = Ult Total Sev / Total Sev@36 mo = $9K / 5K = 1.8$

Ult XS = Total Reported Loss \times Derived Unlim LDF \times Ins Charge@Ult = $(1M + 275K) \times 1.80 \times 0.30 = 688.5K$

Rept XS = Total Report Loss \times Ins Charge@36 mo = $(1M + 275K) \times 0.20 = 255K$

XS IBNR = Ult XS - Rept XS = $688.5K - 255K = 413.5K$

Sample 7

LR approach = $P \cdot E \cdot \chi$ - Rept XS

$$XSLDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{1 - R_{ult}^{500}}{1 - R_{36}^{500}}$$

$$3.00 = LDF_{36-ult}^{unlim}$$

$$R_{ult}^{500} = 6,600 / 9,000 = 0.733$$

$$R_{36}^{500} = 4,500 / 5,000 = 0.900$$

$$LDF_{36-ult}^{unlim} = 3.00 \times 0.1 / 0.267 = 1.125$$

Ult XS = Total Reported Loss \times Derived Unlim LDF \times Ins Charge@Ult = $(1M + 275K) \times 1.125 \times 0.30 = 430.3125K$

Rept XS = Total Report Loss \times Ins Charge@36 mo = $(1M + 275K) \times 0.20 = 255K$

XS IBNR = Ult XS - Rept XS = $430.3125K - 255K = 175.3125K$

Sample 8

LR approach = $P \cdot E \cdot \chi$ - Rept XS

$$XSLDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{1 - R_{ult}^{500}}{1 - R_{36}^{500}}$$

$$3.00 = LDF_{36-ult}^{unlim}$$

$$R_{ult}^{500} = 6,600 / 9,000 = 0.733$$

$$R_{36}^{500} = 4,500 / 5,000 = 0.900$$

$$LDF_{36-ult}^{unlim} = 3.00 \times 0.1 / 0.267 = 1.125$$

Ult XS = Total Reported Loss \times Derived Unlim LDF \times Ins Charge@Ult = $(1M + 275K) \times 1.125 \times 0.30 = 430.3125K$

Rept XS = 275K (Given)

XS IBNR = Ult XS - Rept XS = $430.3125K - 275K = 155.3125K$

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 9

LR approach = $P \cdot E \cdot \chi$ - Rept XS

$$LDF_{36-ult}^{unlim} = 9K / 5K = 1.80$$

$$R_{ult}^{500} = 6,600 / 9,000 = 0.733$$

$$R_{36}^{500} = 4,500 / 5,000 = 0.900$$

$$XSLDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{1 - R_{ult}^{500}}{1 - R_{36}^{500}}$$

$$LDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{R_{ult}^{500}}{R_{36}^{500}}$$

$$XSLDF_{36-ult} = 1.80 \times (1 - 0.733) / (1 - 0.900) = 4.80$$

$$LDF_{36-ult} = 1.80 \times 0.733 / 0.900 = 1.467$$

$$\text{Ult XS Loss} = 275K \times 4.8 = 1,320K$$

$$\text{Ult Lim Loss} = 1M \times 1.467 = 1,467K$$

$$\text{Ult Total Loss} = \text{Ult XS Loss} + \text{Ult Lim Loss} = 1,320K + 1,467K = 2,787K$$

$$\text{Ult XS} = \text{Ult Total Loss} \times \text{Ins Charge@Ult} = 2,787K \times 0.30 = 836K$$

$$\text{Rept XS} = (1,000K + 275K) \times 0.20 = 255K$$

$$\text{XS IBNR} = \text{Ult XS} - \text{Rept XS} = 836K - 275K = 561K$$

Sample 10

LR approach = $P \cdot E \cdot \chi$ - Rept XS

$$\text{Excess Ratio} = 275K / 1,275K = 0.216$$

$$\text{Ult XS} = \text{Ult Total Sev} \times \text{Counts} \times \text{Derived Excess Ratio} = 9,000 \times 1,000 \times 0.216 = 1,941K$$

$$\text{Rept XS} = 275K \text{ (Given)}$$

$$\text{XS IBNR} = \text{Ult XS} - \text{Rept XS} = 1,941K - 275K = 1,666K$$

Sample 11

LR approach = $P \cdot E \cdot \chi$ - Rept XS

$$LDF_{36-ult}^{unlim} = 9K / 5K = 1.80$$

$$R_{ult}^{500} = 6,600 / 9,000 = 0.733$$

$$R_{36}^{500} = 4,500 / 5,000 = 0.900$$

$$LDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{R_{ult}^{500}}{R_{36}^{500}}$$

$$LDF_{36-ult} = 1.80 \times 0.733 / 0.900 = 1.467$$

$$\text{Ult XS Loss} = \text{Rept Lim Loss} \times \text{Derived Limited LDF} \times \text{Insurance Charge Ult} = 1M \times 1.467 \times 0.30 = 440K$$

$$\text{Rept XS} = \text{Rept Lim Loss} \times \text{Insurance Charge@36 mo} = 1,000K \times 0.20 = 200K$$

$$\text{XS IBNR} = \text{Ult XS} - \text{Rept XS} = 440K - 200K = 240K$$

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 12

$$\text{LR approach} = P \cdot E \cdot \chi$$

$$\text{Exp XS@36 mo} = \text{Total Sev@36 mo} \times \text{Counts} \times \text{Insurance Charge@36 mo} = 5,000 \times 1,000 \times 0.20 = 1\text{M}$$

$$\text{Exp Ultimate XS} = \text{Ultimate Total Sev} \times \text{Counts} \times \text{Insurance Charge Ult} = 9,000 \times 1,000 \times 0.30 = 2.7\text{M}$$

$$\text{XS IBNR} = \text{Exp Ult XS} - \text{Exp XS@36 mo} = 2.7\text{M} - 1\text{M} = 1.7\text{M}$$

Sample 13

$$\text{LR approach} = P \cdot E \cdot \chi$$

$$\text{Exp XS@36 mo} = \text{Limited Sev@36 mo} \times \text{Counts} \times \text{Insurance Charge@36 mo} = 4,500 \times 1,000 \times 0.20 = 900\text{K}$$

$$\text{Exp Ultimate XS} = \text{Ultimate Limited Sev} \times \text{Counts} \times \text{Insurance Charge Ult} = 6,600 \times 1,000 \times 0.30 = 1,980\text{K}$$

$$\text{XS IBNR} = \text{Exp Ult XS} - \text{Exp XS@36 mo} = 1,980\text{K} - 900\text{K} = 1,080\text{K}$$

Sample 14

$$\text{Ult XS} = \text{Ult Total Sev} \times \text{Counts} \times \text{Ins Charge @ Ult} = 9,000 \times 1,000 \times 0.3 = 2.7\text{M}$$

$$\text{XS IBNR} = \text{Ult XS} - \text{Rept XS} = 2.7\text{M} - 275\text{K} = 2,425\text{K}$$

Sample 15

$$XSLDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{1 - R_{ult}^{500}}{1 - R_{36}^{500}}$$

$$XSLDF_{36-ult} = 1.8 \times (1 - 0.733) / (1 - 0.9) = 4.8$$

$$\text{XS IBNR} = 275\text{K} \times (4.8 - 1) = 1,045\text{K}$$

Sample 16

$$\text{XS Ultimate} = \text{Ultimate Total Loss} \times \text{XS Ratio} + \text{Ultimate Total Loss} \times (1 - \text{XS Ratio}) \times \text{Aggregate Ratio}$$

$$\text{XS Ultimate} = 9,000 \times 1,000 \times (1 - 6,600 / 9,000) + 9,000 \times 1,000 \times (6,600 / 9,000) \times 0.3 = 4.38\text{M}$$

$$\text{XS IBNR} = \text{XS Ultimate} - \text{Rept XS} = 4.38\text{M} - 275\text{K} = 4.105\text{M}$$

Sample 17

$$275\text{K} = \text{Expected Loss@36 mo} \times \text{Insurance Charge@36 mo}$$

$$\text{Expected Loss@36 mo} = 275\text{K} / 0.2 = 1,375\text{K}$$

$$\text{Ultimate Total Loss} = 1,375\text{K} \times \text{Unlimited LDF} = 1,375\text{K} \times 9,000 / 5,000 = 2,475\text{K}$$

$$\text{Ultimate XS Loss} = \text{Ultimate Total Loss} \times \text{Insurance Charge Ult} = 2,475\text{K} \times 0.3 = 742.5\text{K}$$

$$\text{XS IBNR} = \text{Ultimate XS Loss} - \text{Rept XS} = 742.5\text{K} - 275\text{K} = 467.5\text{K}$$

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 18

LR approach = $P \cdot E \cdot \chi - \text{Rept XS}$

$$XSLDF_{36-ult} = LDF_{36-ult}^{unlim} \times \frac{1 - R_{ult}^{500}}{1 - R_{36}^{500}}$$

$$3.00 = LDF_{36-ult}^{unlim}$$

$$R_{ult}^{500} = 6,600 / 9,000 = 0.733$$

$$R_{36}^{500} = 4,500 / 5,000 = 0.900$$

$$LDF_{36-ult}^{unlim} = 3.00 \times 0.1 / 0.267 = 1.125$$

$$\text{Ultimate Total Loss} = (1,000K + 275K) \times 1.125 = 1,434.375K$$

$$\text{XS Ratio} = 1 - 6,600 / 9,000 = 0.267$$

$$\text{Ultimate XS Loss} = 1,434.375K \times 0.267 = 382.5K$$

$$\text{XS IBNR} = \text{Ultimate XS Loss} - \text{Rept XS} = 382.5K - 275K = 107.5K$$

Sample 19

LR approach = $P \cdot E \cdot \chi - \text{Rept XS}$

$$\text{Ultimate XS Loss} = \text{Ultimate Limited Sev} \times \text{Counts} \times \text{Insurance Charge} = 6,600 \times 1,000 \times 0.3 = 1,980K$$

$$\text{XS IBNR} = \text{Ultimate XS Loss} - \text{Rept XS} = 1,980K - 275K = 1,705K$$

Sample 20

$$\text{Ultimate Limited Loss} = 1,000K \times \text{Derived Limited LDF} = 1,000K \times (\text{Ultimate Limited Sev} / \text{Limited Sev@36 mo}) = 1,000K \times (6,600 / 4,500) = 1,466.67K$$

$$\text{Ultimate XS Loss} = 275K \times \text{Derived XS LDF} = 275K \times (\text{Ultimate XS Sev} / \text{Limited Sev@36 mo}) = 275K \times [(9,000 - 6,600) / (5,000 - 4,500)] = 275K \times 4.8 = 1.32M$$

$$\text{Ultimate Total Loss} = \text{Ultimate Limited Loss} + \text{Ultimate XS Loss} = 1.467M + 1.32M = 2.787M$$

$$\text{Ultimate XS Loss} = \text{Ultimate Total Loss} \times \text{Insurance Charge} = 2.787M \times 0.3 = 836K$$

$$\text{Rept XS} = \text{Total Loss@36 mo} \times \text{Insurance Charge@36 mo} = (1,000K + 275K) \times 0.2 = 255K$$

$$\text{XS IBNR} = \text{Ultimate XS Loss} - \text{Rept XS} = 836K - 255K = 581K$$

Assessment of 2 methods

Sample responses:

- The modeled XS IBNR is less reasonable compared to the LR approach. There may be imperfect data or uncertainty volatile in the actual loss experience.
- The modeled XS IBNR is higher than the LR approach because XS LDFs tend to be highly leveraged, especially in early maturities. And the LR approach doesn't take into consideration of actual experience and uses industry factors, which may not be appropriate. Using credibility approach of the two methods might be more appropriate.
- Modeled XS IBNR is much higher than calculated using LR method. This may be due to the highly leveraged XS LDF that is applied. LR approach is more stable than direct development.

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- LR approach IBNR is much higher than modeled. This is because XS LDF is very volatile and uncertainty in estimation is high. Also we may have little reported XS loss at early maturity for WC which is long tail and subject to latent claims.

Part b: 0.5 point

Sample responses:

- Since the number of claims is expected to go up (ATA of 1.15 and 1.05), that will affect the severity unlimited and limited in different ways.
- By using the limited severity at 36 month, we are assuming that all claims have been reported by 36 month. Since there is still claim count development after 36 month, this violates the assumption and it is not proper to use the limited severity at 36 month.
- Since not all losses are reported at 36 months, the 36 months severity may be overstated, there are more claims expected which may be small.
- The issue is that the expected claims at age 36 is less than 1,000, only $1,000 / (1.15 \times 1.05) = 828$ are reported, likely those unreported are the most complex and severe. The unreported counts will have higher severity than the given severities at ultimate. Using the given severities will understate the IBNR.
- A number of claims have yet to be reported based on the development factors given. Claims that exceeded the deductible are likely to do so after longer periods of time, i.e. there is more development in the excess layer than in the deductible layer.

EXAMINER'S REPORT

Candidates were expected to apply Direct development method and Loss Ratio method (specifically Table M) to calculate the Excess layer IBNR, and assess the appropriateness of using the methods in a long-tailed line such as WC.

Candidates were also expected to understand the shortcomings of the Loss Ratio approach when claim counts are still developing.

Part a

Candidates were expected to calculate the XS IBNR using the Direct Development approach and the LR approach with the Table M (insurance charge) provided (or an alternative approach that calculates valid XS Ultimate Loss and Reported XS Loss), and then determine the most appropriate method to use in the given situation with logical support.

Common mistakes for LR and Modeled IBNR calculations included:

- Only calculating the ultimate loss without calculating the IBNR.
- Treating the ultimate loss as IBNR.
- Applying XS ultimate loss and reported loss to the insurance charge.
- Applying the wrong formula to get the limited/XS LDF when using the severity relativities (e.g., Limited LDF = Unlimited LDF $\times (1 - R_{Ult}) / (1 - R_{36})$, when it should be XS LDF).
- Applying both insurance charges (36 month and ultimate) to the ultimate loss.
- Not applying the insurance charge to the ultimate unlimited/limited loss.
- Applying Limited LDF to the total loss or applying the Unlimited LDF to the limited loss.

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Common mistakes for the assessment of the 2 methods included:

- Only stating that one method is lower/higher than the other method without providing valid reasoning, since numerical support alone was not sufficient (e.g., LR approach is higher than the Modeled approach, and therefore Modeled approach is not reasonable).

Part b

Candidates were expected to know that using counts that are still developing will distort the limited severity calculations or the IBNR calculations.

Common mistakes included:

- Calculating the XS/Limited counts LDF using the severity relativities even though too little information was given, and not tying back to the claim count development's impact on the severity calculation.
- Mentioning other valid issues of the LR approach without tying back to the claim count development, such as adjusting for inflation or different development patterns between excess and limited layer.

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SPRING 2018 EXAM 7, QUESTION 3

TOTAL POINT VALUE: 2

LEARNING OBJECTIVE(S): A1

SAMPLE ANSWERS

Sample 1

$$\text{MSE} = 2,000 \times [z^2/p + 1/q + (1-z)^2 / \sqrt{p}] \times q^2$$

Individual: Z = 1

Collective: Z = 0

MSE Ind = 38.736

MSE Coll = 38.729

Collective is preferred since the MSE is lower.

$$m_1 = (2.5 + 2.1 + 3.0 + 3.5) / (8 + 8.32 + 8.65 + 9) = x = 0.327$$

$$m_2 = (3.335 - 2.5 + 2.705 - 2.1 + 4.113 - 3.0) / (8 + 8.32 + 8.65) = y = 0.102$$

$$m_3 = (3.942 - 3.335 + 3.335 - 2.705) / (8 + 8.32) = z = 0.076$$

$$m_4 = (4.021 - 3.942) / 8 = t = 0.0099$$

$$x + y + z + t = a = \text{ELR} = 0.515$$

$$p_{2015} = (x + y + z) / a = 0.981$$

$$q_{2015} = 1 - p_{2015} = 0.019$$

Sample 2

$$\text{MSE Formula: } E[\alpha^2(U)] \times q^2 \times [z^2/p + 1/q + (1 - z)^2 / t]$$

Incremental Paid Loss

AY	12	24	36	48
2014	2,500	835	607	79
2015	2,100	605	630	
2016	3,000	1,113		
2017	3,500			

$$m_{12} = (2,500 + 2,100 + 3,000 + 3,500) / (8,000 + 8,320 + 8,650 + 9,000) = 0.33$$

$$m_{24} = (835 + 605 + 1,113) / (8,000 + 8,320 + 8,650) = 0.10$$

$$m_{36} = (607 + 630) / (8,000 + 8,320) = 0.08$$

$$m_{48} = 79 / 8,000 = 0.01$$

$$m_{\text{total}} = 0.33 + 0.10 + 0.08 + 0.01 = 0.51$$

$$p_{36} = (0.33 + 0.10 + 0.08) / 0.51 = 0.98$$

$$q_{36} = 1 - p_{36} = 0.02$$

$$t_{36} = \sqrt{0.98} = 0.99$$

For individual LR method, Z = 100%, MSE = 2,000 × (1 - 0.98)² × [1² / 0.98 + 1 / (1 - 0.98) + 0] = 41

For collective LR method, Z = 0%, MSE = 2,000 × (1 - 0.98)² × [0 + 1 / (1 - 0.98) + 1 / 0.99] = 41

Since the MSEs are roughly equal, they are equally preferable.

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Sample 3

$$\text{MSE} = E[\alpha^2(U)] \times q^2 \times [z^2/p + 1/q + (1 - z)^2 / t]$$

$$m_1 = (2,500 + 2,100 + 3,000 + 3,500) / (8,000 + 8,320 + 8,650 + 9,000) = 0.327$$

$$m_2 = (835 + 605 + 1,113) / (8,000 + 8,320 + 8,650) = 0.102$$

$$m_3 = (607 + 630) / (8,000 + 8,320) = 0.076$$

$$m_4 = 79 / 8,000 = 0.01$$

$$m_{\text{total}} = 0.327 + 0.102 + 0.076 + 0.01 = 0.515$$

$$p_2 = (0.327 + 0.102 + 0.076) / 0.51 = 0.981$$

$$q_2 = 1 - 0.981 = 0.019$$

$$t_2 = \sqrt{0.981} = 0.990$$

$$Z_2 \text{ Individual} = 1$$

$$Z_2 \text{ Collective} = 0$$

$$\text{MSE(Ind)} = 2,000 \times [1/0.981 + 1/0.019 + 0] \times 0.019^2 = 38.74$$

$$\text{MSE(Coll)} = 2,000 \times [0 + 1/0.019 + 1/0.99] \times 0.019^2 = 38.73$$

$$Z_2 \text{ Optimal} = p_2 / (p_2 + t) = 0.498$$

Since Z Optimal is closer to 0 it assigns more weight to the Collective method. Therefore, the Collective method is preferable to estimate Z_{2015} .

EXAMINER'S REPORT

Candidates were expected to calculate the inputs needed to determine the mean squared error for the individual loss ratio method and collective loss ratio method.

Specifically, candidates were expected to determine the following:

- Incremental amount of expected paid claims per unit of exposure for each development period.
- Loss ratio payout factor and loss reserve factor.
- The amount of credibility for each method.

Common mistakes included:

- Not knowing the mean squared error formula for the two methods.
- Using the loss ratio payout factor and loss reserve factor from the wrong period.
- Using a method other than the loss ratio method to calculate the payout factor and reserve factor.
- Using the wrong amount of credibility for each method.

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SPRING 2018 EXAM 7, QUESTION 4

TOTAL POINT VALUE: 2

LEARNING OBJECTIVE(S): A1

SAMPLE ANSWERS

Sample 1

$$\bar{x} = \frac{0.6282 + 0.6375 + 0.6941}{3} = 0.6533$$

$$\bar{x}^2 = 0.4268$$

$$\overline{x^2} = \frac{0.6282^2 + 0.6375^2 + 0.6941^2}{3} = 0.4276$$

$$\bar{y} = 0.1126 + 0.6533b$$

$$b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} = \frac{0.5979 - 0.6533\bar{y}}{0.4276 - 0.4268}$$

$$b = \frac{0.5979 - 0.6533 \times (0.1126 + 0.6533b)}{0.0008} = 1.2261$$

For AY 2017,

$$y = 0.1126 + 1.2261x = 0.1126 + 1.2261 \times \frac{6,000,000}{8,000,000} = 1.0322$$

$$Ultimate = 1.0322 \times 8,000,000 = 8,257,600$$

Sample 2

$$\overline{XY} = \bar{X}\bar{Y} = 0.5979 = \frac{1}{3} \times (0.6282 \times \hat{Y}_1 + 0.6375 \times \hat{Y}_2 + 0.6941 \times \hat{Y}_3)$$

$$= \frac{1}{3} \times [0.6282 \times (a + 0.6282b) + 0.6375 \times (a + 0.6375b) + 0.6941 \times (a + 0.6941b)]$$

$$= \frac{1}{3} \times (1.9598a + 1.2828b)$$

$$= \frac{1}{3} \times (1.9598 \times 0.1126 + 1.2828b)$$

$$b = 1.2261$$

$$Ult_{2017} = \left(0.1126 + 1.2261 \times \frac{6,000,000}{8,000,000}\right) \times 8,000,000 = 8,257,600$$

EXAMINER'S REPORT

Candidates were expected to compute the least square estimate by applying and solving the equations for coefficients a and b.

Common mistakes included:

- Not calculating AY 2017 projected ultimate losses.

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- Deriving AY 2017 Projected ultimate loss as a product of AY 2017 projected ultimate loss ratio and AY 2017 incurred loss.
- Estimating \bar{Y} from the graph

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SPRING 2018 EXAM 7, QUESTION 5	
TOTAL POINT VALUE: 2.25	LEARNING OBJECTIVE(S): A1, A2
SAMPLE ANSWERS	
Part a: 1.5 points	
<p><u>Sample 1 – Incremental Paid Formulas</u></p> <p>BF Ultimate – Benktander Ultimate = 100,000</p> <p>BF Ultimate = $850 + x + 4,000 \times (0.675) \times [1 - (3.00/1.60)^{-1}] = 2,110 + x$</p> <p>GB Ultimate = $850 + x + (2,110 + x) \times [1 - (3.00/1.60)^{-1}] = 1,834.67 + 1.467x$</p> <p>$2,110 + x - 1,834.67 - 1.467x = 100$</p> <p>$x = 375.45$</p> <p><u>Sample 2 – Cumulative Paid Formulas</u></p> <p>$U_{GB} + 100,000 = U_{BF}$</p> <p>$24\text{-Ult LDF} = 3.00/1.60 = 1.875$</p> <p>BF Ult = $x + 4,000,000 \times (0.675) \times (1 - 1.875^{-1}) = 1.26M + x$</p> <p>Benktander Ult = $x + (1.26M + x) \times (0.46667) = 1.46667x + 0.588M$</p> <p>$1.26M - 0.1M + x = 1.46667x + 0.588M$</p> <p>$x = 1,225,706$</p> <p>$1,225,706 - 850,000 = 375,706$</p>	
Part b: 0.25 point	
<p><u>Sample Response</u></p> <ul style="list-style-type: none"> • Benktander ultimates would be greater than BF ultimates if the CL Ultimate > BF Ultimate since Benktander is a weighting of the two methods. • Actual paid > expected to date, then the Benktander ultimates are higher • When paid losses emerge higher than expected (in this case, higher than 590k). <ul style="list-style-type: none"> ○ $(2.11 + x) \times (1 - 1/1.875) + 0.85 + x > 2.11 + x$ ○ $x > 0.590$ • When $U_{BF} > U_0$ 	
Part c: 0.5 point	
<p><u>Sample Response</u></p> <ul style="list-style-type: none"> • When losses develop downward, the BF method will keep the forward looking IBNR the same, regardless of how losses to date have performed. Thus, the downward development will not affect IBNR. However, in reality, the downward development may be indicating salvage & subro trends that we would also want to apply to our IBNR. • This method selects a priori loss estimate through judgment. If there have been a significant number of claims than usual, this method completely ignores this, even if reported claims > ultimate claims estimated by this method. In these instances, it seems unreasonable to completely ignore such reported claims. • Losses developing downward is unusual and typically the result of favorable closures on claims. Once a claim is closed, it is generally unlikely to reopen and increase further. BF does not recognize this (assumes losses to date not predictive) when, in fact, downward 	

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development is generally a good prediction that ultimate losses will be less than expected.

EXAMINER'S REPORT

Candidates were expected to calculate unpaid claim estimates using the following loss reserving methods: Budgeted Loss, Chain Ladder, Bornhuetter Ferguson, and Benktander. Candidates were expected to identify and evaluate the strengths and weaknesses and relationships of unpaid claims models at a basic conceptual level by comparing and contrasting these methods, and applying the knowledge of their strengths and weaknesses of these methods in a provided scenario.

Part a

Candidates were expected to calculate unpaid claim estimates using the following loss reserving methods: Budgeted Loss, Chain Ladder, Bornhuetter Ferguson, and Benktander.

Common mistakes included:

- Selecting an incorrect Link Ratio (deriving the 24-ultimate incorrectly or using the 12-to ultimate).
- Setting up the relationship between the UBF and UGB as given in the problem (e.g. using the wrong sign on the 100k portion of the equation, or dropping the 100k from the relationship altogether).
- Reversing the $p = \% \text{Reported}$ vs. $q = \% \text{Unreported}$ in the calculation of ultimates.

Part b

Candidates were expected to identify and evaluate the strengths and weaknesses and relationships of unpaid claims models at a basic conceptual level.

Common mistakes included:

- Requiring a minimum amount of weight on the Chain Ladder method (e.g. $p = \% \text{reported}$ would need to be greater than 50%).
- Stating that losses “come in higher” without a reference comparing to what (e.g. “come in higher than expected”).

Part c

Candidates were expected to know the strengths and weaknesses of the B-F method and justify why it may not be appropriate for the situation in the provided scenario.

Common mistakes included:

- Not being clear that the BF Reserve or IBNR is independent of the claim emergence. Many candidates stated or implied that the ultimate was independent when in fact claim emergence is a portion of the ultimate calculation.
- Citing as facts that negative IBNR is not possible or that $\% \text{Reported}$ cannot exceed 100%, when in fact this is a possible occurrence (e.g. subrogation or salvage at the end of a property damage claim).

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- Justifying the better method with a generalized phrase without providing an underlying rationale or example (e.g. “we should reflect actual experience more” or “we should rely upon actual experience less”).

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SPRING 2018 EXAM 7, QUESTION 6	
TOTAL POINT VALUE: 3	LEARNING OBJECTIVE(S): A2, A3
SAMPLE ANSWERS	
Part a: 1.5 points	
<u>Sample 1</u>	
Reserve = Prem × ELR × (1 – lag) = Prem × ELR × [1 – G(x)]	
1 – G(x) = $\Theta / (x + \Theta)$	
400 = 1,000 × ELR × [$\Theta / (42 + \Theta)$]	
553.85 = 1,200 × ELR × [$\Theta / (30 + \Theta)$]	
0.4 × (42 + Θ) = ELR × Θ	
0.4615 × (30 + Θ) = ELR × Θ	
0.4 Θ + 16.8 = 13.845 + 0.4615 Θ	
2.955 = 0.0615 Θ	
$\Theta = 48.05$	
ELR = 0.4 × (42 + 48.05) / 48.05 = 0.75	
<u>Sample 2</u>	
Reserves = Adj. Prem × ELR × (1 – G(x))	
1,133.33 / 818.18 = [1,700 × ELR × (1 – G(6))] / [1,500 × ELR × (1 – G(18))]	
1.385 = 1.133 × [(1 – G(6)) / (1 – G(18))]	
1.222 = [$\Theta / (6 + \Theta)$] × [(18 + Θ) / Θ]	
7.332 + 1.222 $\Theta = 18 + \Theta$	
0.222 $\Theta = 10.668$	
$\Theta = 48.05$	
G(42) = 42 / (42 + 48.05) = 0.466	
G(30) = 0.384	
G(18) = 0.273	
G(6) = 0.111	
ELR = (275 + 306 + 344 + 220) / (1,000 × 0.466 + 1,200 × 0.384 + 1,500 × 0.273 + 1,700 × 0.111)	
ELR = 0.751	
<u>Sample 3</u>	
1,000 × ELR × [1 – G(42)] = 400	
1,200 × ELR × [1 – G(30)] = 553.85	
1,500 × ELR × [1 – G(18)] = 818.18	
1,700 × ELR × [1 – G(6)] = 1,133.33	
0.4 = ELR × [1 – 42 / (42 + Θ)]	
0.4615 = ELR × [1 – 30 / (30 + Θ)]	

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$$1.15375 = [1 - 30 / (30 + \Theta)] / [1 - 42 / (42 + \Theta)]$$

$$1.15375 = [\Theta / (30 + \Theta)] \times [(42 + \Theta) / \Theta]$$

$$1.15375 = (42 + \Theta) / (30 + \Theta)$$

$$1.15375 \times (30 + \Theta) = 42 + \Theta$$

$$34.6125 + 1.15375 \Theta = 42 + \Theta$$

$$0.15375 \Theta = 7.3875$$

$$\Theta = 48$$

$$0.4 = \text{ELR} \times [1 - 42 / (42 + 48)]$$

$$0.4 = \text{ELR} \times 0.5333$$

$$\text{ELR} = 0.75$$

Sample 4

$$\text{ELR} = \text{Paid Loss} / \text{Used-up Prem} = 1,145 / p \quad (p \text{ equals used-up premium})$$

$$\text{On Level Prem} = 1,000 + 1,200 + 1,500 + 1,700 = 5,400$$

$$\text{Total Reserves} = 400 + 553.85 + 818.18 + 1,133.33 = 2,905.36 = \text{ELR} \times (5,400 - p)$$

$$= 2,905.36 / (5,400 - p)$$

$$1,145 / p = 2,905.36 / (5400 - p)$$

$$1,145 \times 5,400 - 1,145p = 2,905.36p$$

$$1,145 \times 5,400 = 4,050.36p$$

$$1,526.5 = p$$

$$\text{ELR} = 1,145 / 1,526.5 = 75\%$$

Part b: 0.75 point

Sample 1

AY	Avg Age	(1) G(x)	(2) Cum Pd Loss	(3) = (2)/(1) Ultimate Loss	(4) Premium	(3)/(4) Ultimate Loss Ratio
'14	42	0.467	275	588.865	1,000	0.589
'15	30	0.385	306	794.805	1,200	0.662
'16	18	0.273	344	1,260.073	1,500	0.840
'17	6	0.111	220	1,981.928	1,700	1.166

↓ upward trend

The loss ratio is increasing from AY to the next. It is not appropriate to use Cape Cod.

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Sample 2

Testing for ELR constancy

AY	Avg Age	G(x)	Used-up Premium	LR
2014	42	0.4668	466.77	0.5892
2015	30	0.3847	461.657	0.6628
2016	18	0.2728	409.215	0.8406
2017	6	0.1112	188.959	1.1643

$\Theta = 47.98$ (from part a)

$G(x) = x / (x + \Theta)$

$G(42) = 42 / (42 + 47.98) = 0.4668$

Used up prem = On Level Prem \times G(x)

LR = Cumulative Paid loss (given) / used up prem

The ELR does not seem appropriate because the loss ratios (LR) are increasing indicating that reserves will be overstated in older years and understated in more recent years.

Part c: 0.75 point

Sample 1

Process variance – the variance associated with the random chance associated with modelling the insurance process

Parameter variance – the variance associated with the chance that the parameters of our model are incorrect

Parameter variance can be reduced by limiting the number of parameters in our model.

Sample 2

Process variance – risk from the randomness of insurance business

Parameter risk – risk that estimated parameters do not correctly predict insurance losses, and don't fully account for uncertainty

To reduce parameter risk, use Cape Cod over LDF method because Cape Cod uses additional information, thus less parameters

Sample 3

Process variance – due to the pure randomness of the valuation

Parameter variance – due to that we can't exactly estimate the parameters

Reduce parameter variance: try to reduce parameter's number. The fewer parameters, we will have smaller parameter variance

EXAMINER'S REPORT

Candidates were expected to know how to calculate the expected loss ratio used in the Cape Cod method, and be able to evaluate the appropriateness of using the Cape Cod method. They were also expected to know the two types of variance and know the basics behind both.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part a

Candidates were expected to know the formula for estimated reserve. Given this knowledge, the question provided information to set up four equations for the estimated reserve by accident year. Using two of these equations there would be two unknowns (Theta and ELR). The candidate then had two ways to solve for these: 1. Divide one equation by the other to eliminate one of the variables (presumably ELR), or 2. Substitute one equation into the other (presumably solve for ELR in one equation and substitute it into the other equation). Once you solve for Theta, you can use one of these two equations to solve for ELR.

Common mistakes included:

- Not knowing how to solve for the two unknowns (Theta and ELR) once the equations had been set up, or using a method of solving which was accurate but perhaps more time consuming (e.g. quadratic equation).
- Calculating $G(x)$ as Paid Loss / (Paid Loss + Estimated Reserve) even though the problem stated $G(x) = x / (x + \Theta)$ where x represents the average age of paid losses.

Part b

Candidates were expected to know how to translate $G(x)$ into either expected ultimate loss using the LDF method or used up premium, and then see if there was a trend in loss ratios to evaluate the appropriateness of using a constant Expected Loss Ratio across all accident years for this book of business.

The most common error was adding the Cape Cod reserve to paid loss to calculate the ultimate loss for each accident year. Candidates were expected to recognize that they were asked to evaluate the appropriateness of using a constant ELR for all accident years combined and that the given reserves were already estimated with the Cape Cod method, using the constant ELR from part a.

Other mistakes included:

- Stating that the Cape Cod method is not appropriate because the premium volume is increasing.
- Not using the growth function provided in the question.

Part c

Candidates were expected to know basic information about both process variance and parameter variance and be able to give a definition of each.

Common mistakes included:

- Only listing the two types of variance without including a definition.
- Saying that using fewer parameters will reduce process variance.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 7										
TOTAL POINT VALUE: 2.5	LEARNING OBJECTIVE(S): A2									
SAMPLE ANSWERS										
Part a: 0.5 point										
<p><u>Sample 1</u></p> $36-48 = \frac{1.043 \times 3,500^2 + 1.158 \times 5,050^2}{3,500^2 + 5,050^2} = 1.121$ <p>48-Ult = 1.027</p> <p>2014 Ultimate = 3,700 × 1.121 × 1.027 = 4,260</p> <p><u>Sample 2</u></p> $\text{LDF } 36-48 = \frac{3,500 \times 3,650 + 5,050 \times 5,850}{3,500^2 + 5,050^2} = 1.121$ $\text{LDF } 48-60 = \frac{3,650 \times 3,750}{3,650^2} = 1.027$ <p>LDF 36-Ult = 1.121 × 1.027 = 1.151</p> <p>Ultimate loss = 3,700 × 1.151 = 4,258.7</p> <p><u>Sample 3</u></p> <p>Use least squares LDF</p> <table style="margin-left: 40px;"> <tr> <td></td> <td style="text-align: center;"><u>36-48</u></td> <td style="text-align: center;"><u>48-60</u></td> </tr> <tr> <td>LDF</td> <td style="text-align: center;">1.121</td> <td style="text-align: center;">1.027</td> </tr> <tr> <td>CDF</td> <td style="text-align: center;">1.151</td> <td style="text-align: center;">1.027</td> </tr> </table> <p>AY 2014 Ultimate = 3,700K × 1.151 = 4,260K</p>			<u>36-48</u>	<u>48-60</u>	LDF	1.121	1.027	CDF	1.151	1.027
	<u>36-48</u>	<u>48-60</u>								
LDF	1.121	1.027								
CDF	1.151	1.027								
Part b: 0.5 point										
<p><u>Sample 1</u></p> $f_{36-48} = \frac{1.043 \times 3,500 + 1.158 \times 5,050}{3,500 + 5,050} = 1.1109$ <p>AY 2014 Ult = 3,700 × 1.1109 × 1.027 = 4,221.4</p> <p><u>Sample 2</u></p> $\frac{3,650 + 5,850}{3,500 + 5,050} = 1.111$ <p>3,700 × 1.111 × 1.027 = 4,222</p>										

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 3

Use Volume Weighted LDF

	<u>36-48</u>	<u>48-60</u>
LDF	1.111	1.027
CDF	1.141	1.027

AY 2014 Ultimate = 3,700K × 1.141 = 4,222K

Part c: 0.5 point

Sample 1

$$f_{36-48} = \frac{1.043 + 1.158}{2} = 1.101$$

$$3,700 \times 1.101 \times 1.027 = 4,184$$

Sample 2

Use Straight Average LDFs

	<u>36-48</u>	<u>48-60</u>
LDF	1.101	1.027
CDF	1.131	1.027

AY 2014 Ultimate = 3,700K × 1.131 = 4,182K

Part d: 1 point

- To test the variance assumptions, we can plot weighted residuals against $c_{i,k}$
 - Plot a: $(c_{i,k+1} - c_{i,k} \times f_{k0})$ against $c_{i,k}$
 - Plot b: $(c_{i,k+1} - c_{i,k} \times f_{k1}) / \sqrt{c_{i,k}}$ against $c_{i,k}$
 - Plot c: $(c_{i,k+1} - c_{i,k} \times f_{k2}) / c_{i,k}$ against $c_{i,k}$

Compare plots a, b, and c for several values of k to see which shows the most random pattern. That one is the most reasonable for the data in the triangle.

- Calculate the following residuals:
 - Situation a: $r = (c_{i,k+1} - c_{i,k} \times f_{k0})$
 - Situation b: $r = (c_{i,k+1} - c_{i,k} \times f_{k1}) / \sqrt{c_{i,k}}$
 - Situation c: $r = (c_{i,k+1} - c_{i,k} \times f_{k2}) / c_{i,k}$
 Plot these r's against previous cumulative losses. If an assumption's r's are random around 0 with no trends or patterns, then that assumption is appropriate. If all three plots are random, choose f_{k1} because the other two are not superior.

EXAMINER'S REPORT

Candidates were expected to know which Chain Ladder weighting method corresponded to each of the Mack variance assumptions. They were also expected to know how to test which variance assumption was most appropriate.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part a

Candidates were expected to use Mack's Least Squares methodology to generate LDFs. They were also expected to apply the LDFs to generate ultimate losses.

Common mistakes included:

- Using the Brosius Least Squares method.
- Using the simple average variance assumption.
- Using the wrong AY to calculate LDFs.

Part b

Candidates were expected to use the volume weighted average methodology to generate LDFs. They were also expected to apply the LDFs to generate ultimate losses.

Common mistakes included:

- Using an incorrect variance assumption or a different weighting method which wasn't appropriate for the given situation.

Part c

Candidates were expected to use the simple average methodology to generate LDFs. They were also expected to apply the LDFs to generate ultimate losses.

Common mistakes included:

- Using the Least Squares variance assumption, since similar to an OLS regression, Least Squares assumes that variance of the next incremental loss is constant (i.e. variance does not change based on the losses reported to date).
- Using square root or $3/2$ power to weight.

Part d

Candidates were expected to be able to explain Mack's method for testing the proper variance assumption. This appeared to be the challenging part of the question for candidates.

Common mistakes included:

- Describing the linearity test instead.
- Describing a method that did not test the variance assumption (e.g. comparing MSE or adjusted SSE).
- Comparing the values in the triangle for seasonality or outliers.
- Not providing sufficient detail and/or inaccurate detail.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 8

TOTAL POINT VALUE: 2.75

LEARNING OBJECTIVE(S): A2

SAMPLE ANSWERS

Part a: 2 points

Sample 1

$$S_k = \sum (\text{rank}_k - \text{rank}_{k-1})^2; T_k = 1 - 6 \times \frac{S_k}{n \times (n^2 - 1)}; W_k = \#AY - k - 1$$

k=2, rank

<u>1-2</u>	<u>2-3</u>
1	1
3	3
2	2

$$S_2 = 0; T_2 = 1; W_2 = 5 - 2 - 1 = 2$$

k=3, rank

<u>2-3</u>	<u>3-4</u>
1	2
2	1

$$S_3 = 1^2 + 1^2; T_3 = 1 - \frac{2}{2 \times \frac{3}{6}} = -1; W_3 = 1$$

$$T = \frac{\sum T_k \times W_k}{\sum W_k} = \frac{1 \times 2 + (-1) \times 1}{2 + 1} = 0.333$$

$$\text{Var}(T) = \frac{1}{\frac{(\#AY - 2) \times (\#AY - 3)}{2}} = \frac{1}{\frac{(5 - 2) \times (5 - 3)}{2}} = 0.333$$

$$\text{C.I. at 50\%} = (\sqrt{0.333} \times (-0.67), \sqrt{0.333} \times 0.67) = (-0.387, 0.387)$$

Since T = 0.333 falls into C.I. at 50%, do not reject null hypothesis. Hence, no correlation is observed.

Sample 2

AY	r ₁	r ₂	s ₂	r ₃	s ₃
1	1	1	1	1	2
2	4	3	3	2	1
3	3	2	2		
4	2				

l = 5

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

$$T = 1 - 6 \times \frac{\sum(r - s)^2}{n^3 - n}$$

$$T_2 = 1 \text{ (because perfect correlation, positive)}$$

$$T_3 = -1 \text{ (because perfect correlation, negative)}$$

$$T_{global} = \frac{2 \times 1 + 1 \times (-1)}{3} = 0.\bar{3}$$

$$E(T) = 0, Var(T) = \frac{1}{\frac{(I-3) \times (I-2)}{2}} = \frac{1}{\frac{(5-3) \times (5-2)}{2}} = 1/3$$

$$interval = (\sqrt{0.333} \times (-0.67), \sqrt{0.333} \times 0.67) = (-0.3866, 0.3866)$$

T = 1/3 is between (-0.3866, 0.3866), so columns of development do not exhibit correlation.

Sample 3

AY	r ₁	r ₂	s ₂	r ₃	s ₃
1	1	1	1	1	2
2	4	3	3	2	1
3	3	2	2		
4	2				

$$I = 5$$

$$T_2 = 1 - 6 \times \frac{\sum(r_2 - s_2)^2}{3^3 - 3} = 1$$

$$T_3 = 1 - 6 \times \frac{1^2 + 1^2}{2^2 - 2} = -1$$

$$T = \frac{2 \times 1 + 1 \times (-1)}{(5 - 2 - 1) + (5 - 3 - 1)} = \frac{1}{1 + 2} = 0.333$$

$$E(T) = 0, Var(T) = \frac{1}{3 \times \frac{2}{2}} = 0.333$$

$$interval = (\sqrt{0.333} \times (-0.67), \sqrt{0.333} \times 0.67) = (-0.3866, 0.3866)$$

$$C.I. = \pm 0.67 \times \sqrt{\frac{1}{3}} = \pm 0.3868$$

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

0.33 Falls within the CI, so we conclude that there is not significant correlation at 50% level.

Sample 4

$$I = 5 = \#AY; S_k = \sum (\text{rank} - \text{rank}_{\text{prior}})^2; T_k = 1 - \frac{S_k}{n \times \frac{(n^2 - 1)}{6}}; Wt = I - k - 1$$

	2-2	3-4
k	2	3
S_k	0	2
T_k	1	-1
n	3	2
wt	2	1

$$T = \frac{\sum wt \times T_k}{\sum wt} = \frac{2 \times 1 + 1 \times (-1)}{2 + 1} = 0.333$$

$$E(T) = 0, Var(T) = \frac{1}{\frac{(I-2) \times (I-3)}{2}} = 0.333$$

$$CI = E(T) \pm \sqrt{V(T)} \times 0.67 = (-0.387, 0.387)$$

T = 0.333 is within interval, no subsequent correlation exists.

Sample 5

<u>AY</u>	<u>1-2 rank</u>	<u>2-3 rank</u>
1	1	1
2	3	3
3	2	2

Perfectly correlated $\Rightarrow \rho_1 = 1$

<u>AY</u>	<u>1-2 rank</u>	<u>2-3 rank</u>
1	1	2
2	2	1

Perfect negative correlation $\Rightarrow \rho_2 = -1$

Combine, weighted by # obs - 1

$$\rho_{\text{overall}} = \frac{2 \times 1 + 1 \times (-1)}{3} = 0.333$$

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

$$\text{Var}(\rho_{\text{overall}}) = \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)^2 \times 1 = 0.333, E(\rho_{\text{overall}}) = 0$$

$$Z = \frac{\rho_{\text{overall}} - E(\rho_{\text{overall}})}{\sqrt{\text{Var}(\rho_{\text{overall}})}} = \frac{0.333 - 0}{\sqrt{0.333}} = 0.5774$$

Since Z is in (-0.67, 0.67), we do not reject the hypothesis that columns are uncorrelated.

Part b: 0.5 point

Sample Response for avoiding an accumulation of error probabilities

- Avoid an accumulation of error probabilities.
- It avoids the accumulation of error probabilities. In other words, we have a smaller chance of making a type 1 error.
- It's not unusual to see correlation in a few pairs of columns, could be due to randomness but doesn't necessarily mean the whole triangle has correlation.
- Naturally there is a chance that a few columns happen to appear correlated, we cannot say if it is really a correlation or because of randomness.
- Reduce the probability of making errors.
- Due to randomness, we may have some pairs of LDFs exhibiting correlation although overall they are not correlated.
- For each pair, even at 90% confidence level there is still a 10% chance that the pair randomly shows correlation.
- Given a large triangle, there will be correlation between columns by pure chance.

Sample Response for the importance of knowing whether correlations globally prevail

- More important to know whether correlations globally prevail than to find a small part of the triangle with correlations.
- The whole triangle is more important than a pair of columns.
- We focus more on the total unpaid or unreported losses confidence interval, so we should focus more on the entire triangle.
- Because we use the triangle as a whole for valuation.
- Correlation in the whole triangle is more meaningful when doing analysis.
- We care about whether there is development factor correlation as a whole rather than a localized region.
- It's better to account for correlation in the entire dataset than in a small section of the dataset.
- Testing the triangle as a whole gives an overview of the correlations exhibited in the data, which makes more intuitive sense and provides an overall conclusion.

Part c: 0.25 point

Sample Response

- The expected cumulative losses in the next development period is proportional to losses up-to-date.
- Incremental losses in a period only depend on cumulative losses in a prior period.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

- $E[C_{i,k+1} | C_{i,1}, \dots, C_{i,k}] = C_{i,k} \times f_k$
- $E[C_{i,k+1} | data] \propto C_{i,k}$
- Accident years are independent.
- $C(i,k)$ and $C(j,l)$ are independent for all $i \neq j$ and $k \neq l$.
- Development years and accident years are independent.

EXAMINER'S REPORT

Candidates were expected to understand the linearity assumption of the chain ladder method, how to perform a test of the assumption using Spearman correlation, and why it is more appropriate to test the entire triangle globally rather than all column pairs individually.

Part a

Candidates were expected to perform a correlation test using Spearman correlation coefficients and draw a conclusion as to whether subsequent columns of development factors exhibited correlation. This included:

- Calculating the Spearman correlation coefficients between the first two pairs of development factors.
- Taking a weighted average of these correlation coefficients to serve as a test statistic.
- Calculating the confidence interval for the test statistic under the null hypothesis that columns are uncorrelated, which requires calculating the variance of the test statistic and knowing that the expected value of the test statistic is zero.
- Comparing the test statistic to the confidence interval to draw a conclusion on the null hypothesis, and stating the hypothesis being tested.

Common mistakes included:

- Evaluating the correlation between each column pair individually instead of deriving a single test statistic for the triangle, as the syllabus material stresses that simply testing pairs individually is suboptimal.
- Performing a test that relies on Pearson correlation coefficients even though the problem explicitly states the use of Spearman correlation coefficients.
- Using the wrong value for l .
- Skipping the variance calculation and using the standard normal variate directly without normalizing the test statistic.
- Using an incorrect equation for the confidence interval of T (e.g. multiplying 0.67 by the variance of T instead of the standard deviation of T , or using 0.5 from the confidence interval instead of the standard normal variate of 0.67).
- Errors in ranking the development factors.
- Misstating the conclusion, including stating that the null hypothesis was rejected instead of not rejected or that the null hypothesis was that factors are correlated.
- Performing the wrong test, including a calendar year effects test or a correlation test based on Pearson correlation coefficients.
- Simply stating "do not reject null" as the conclusion without stating what the null hypothesis is or whether this means that correlation is/isn't present.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part b

Candidates were expected to describe two reasons why it is more appropriate to test the triangle as a whole, rather than individually testing all pairs in the triangle. Candidates were expected to provide reasons in line with two general themes:

- Testing all pairs individually creates an accumulation of type 1 error probabilities.
- It is more important to know whether correlations globally prevail than to find a small part of the triangle with correlations.

Common mistakes included:

- Providing a second reason that was a rewording of the first.
- Arguing against testing a *single column pair* versus the whole triangle, rather than against testing *pairs of columns* versus the whole triangle.
- Stating that correlation may exist between non-subsequent columns, since part (b) of the question asks about “correlation between pairs of columns” and does not restrict this to subsequent pairs.
- Providing issues that would not be addressed by a development factor correlation test (individually or globally) and would require a different test, such as calendar year effects.
- Providing causes of correlation of calendar year effects such as “larger developments tend to be followed by smaller developments” or “a change in reserving practices or settlement rates” rather than a reason for performing a test on the triangle as a whole instead of testing pairs individually.

Part c

Candidates were expected to know that the chain ladder method assumption being evaluated is the linearity assumption (i.e. expected claims in a period only depend on the previous level of cumulative claims and a factor based on age).

However, it was noted that there is a conflict in the syllabus material on this subject, with some of the material stating that a development factor correlation test evaluates the assumption of accident year independence. As a result, candidates received credit for either response.

Common mistakes included:

- Providing the variance assumption of the chain ladder method.
- Simply stating “the linearity assumption” without elaboration to demonstrate understanding of what the assumption entails.
- Stating that the test in part a. evaluates the assumption that development factors are uncorrelated, as this is an implication of the linearity assumption of the chain ladder method rather than its own assumption.
- Writing an incomplete mathematical expression without adding additional verbal description to complete the assumption (e.g. simply writing “ $E[C_{k+1}] = C_k \times f_k$ ”, as this omits that the expected value is conditional on the previous level of cumulative claims).

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 9	
TOTAL POINT VALUE: 1.25	LEARNING OBJECTIVE(S): A2, A3
SAMPLE ANSWERS	
Part a: 0.75 point	
<p><u>Sample responses for supporting the actuary's claim that the all-year weighted average is inappropriate</u></p> <ul style="list-style-type: none"> • The age to age factor is not stable over time. • The 5-year moving average is not stable. • The age to age factor shows an increasing trend. • There has been a significant change in the age to age factor over time. • The age to age factor has been higher in recent years. • There is a clear shift in the graph. • There is a change in the level around which the age to age factor is fluctuating. <p><u>Sample responses for providing two methods that could improve the estimated age to age factor</u></p> <ul style="list-style-type: none"> • Use a weighted average with more weight on recent years. • Use a 5-year weighted average. • Use the 5-year moving average. • Use a 5-year simple average. • Use a 5-year ex hi/lo average. • Group accident years. • Exclude the accident years where the age to age factors are lower. • Use the state-space model • Fit a curve to age to age factors. • Use a time series analysis. • Use expert opinion to select the age to age factor. • Use industry data to select the age to age factor. • Adjust the data for systematic changes. • Use Berquist-Sherman method to adjust data. • Use a Bayesian method which incorporates expert opinion. • Use ODP GLM on the latest 5 diagonals. • Use CCL model. • Use CSR model. • Compare paid age to age factors to reported age to age factors. 	
Part b: 0.5 point	
<ul style="list-style-type: none"> • Plot residuals against time. If there are strings of positives and negatives then development factors are not stable. • Use the state-space model which compares the volatility of observations around the mean to the volatility in the mean itself over time. • Look for changes in the settlement rate of claims by measuring the percentage of closed claims closed by age. 	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT

Candidates were expected to have knowledge of the six testable implications of the assumptions needed for the chain ladder method to be optimal. This question addressed the fourth testable implication discussed in the Venter text: stability of loss development factors. Candidates were expected to recognize instability by analyzing the graph provided, suggest methods that can be used when instability is present, and discuss a method of testing for instability.

Part a

Candidates were expected to identify the instability in the age to age factor and provide two methods that could be an improvement over the all-year weighted average given the trend in recent accident years. Responses that only indicated volatility in the age to age factor did not receive credit as random fluctuation could actually be an argument for using the all-year weighted average.

Common mistakes included:

- Listing only one method to improve the estimated age to age factor.
- Providing support for using the all-year weighted average.
- Listing two alternate methods without providing any support for the actuary's claim.

Part b

Candidates were expected to describe a method for testing stability discussed in the Venter text other than looking for trends in the moving average.

Common mistakes included:

- Describing other testable implications discussed in the Venter text such as significance or linearity.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 10

TOTAL POINT VALUE: 2.5

LEARNING OBJECTIVE(S): A3

SAMPLE ANSWERS

Part a: 2 points

Sample 1

x	y	\hat{y}	$ \epsilon $
600	1,300	1,375	75
650	1,400	1,400	0
550	1,400	1,350	50
700	1,450	1,425	25

$$\sum(x_i - \bar{x})^2 = 12,500$$

$$s_b = [\sum (\epsilon_i^2 / 25,000)]^{0.5} = [(1 / 25,000) \times (75^2 + 50^2 + 25^2)]^{0.5} = 0.592$$

$$\sum x_i^2 = 1,575,000$$

$$s_a = s_b \times (1 / 4 \times \sum x_i^2)^{0.5} = 0.592 \times (393,750)^{0.5} = 372$$

$s_b > b / 2 \rightarrow$ insignificant

$s_a < a / 2 \rightarrow$ significant

Assumption: the variance of expected future losses in the next development period are proportional to losses to date.

Sample 2

AY	x	Predicted	Actual	$\epsilon=A$ -Predicted
13	600	1,375	1,300	-75
14	650	1,400	1,400	0
15	550	1,350	1,400	50
16	700	1,425	1,450	25

$$Y = a + bx$$

$$a = 1,075$$

$$b = 0.5$$

$$\bar{x} = 625$$

$$\sum x_i^2 = 1,575,000$$

$$\sum(x_i - \bar{x})^2 = 12,500$$

$$S_b = \sqrt{\frac{1}{2 \times (12,500)} \times (75^2 + 50^2 + 25^2)} = 0.592$$

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$$S_a = 0.592 \times \sqrt{\frac{1,575,000}{4}} = 371.5$$

$a = 1,075 > 2 \times S_a = 2 \times 371.5 = 743$; a is significant

$b = 0.5 < 2 \times S_b = 2 \times 0.592$; b is not significant

Assumption: to validate if incremental losses in next period is proportional to the losses reported to date.

Sample 3

Chain ladder assumption tested: expected loss of next development period is proportional to loss-to-date.

$$S_b = \sqrt{\frac{\sum \varepsilon_i^2}{(n-2)\sum(x_i - \bar{x})^2}}$$

$$\bar{x} = 625$$

$$n = 4$$

$$\sum(x_i - \bar{x})^2 = 12,500$$

AY	\hat{Y}_{24}	ε
2013	1,375	-75
2014	1,400	0
2015	1,350	50
2016	1,425	25

$$S_b = \sqrt{\frac{8,750}{(2) \times 12,500}} = 0.592$$

$0 > b - 1.96 \times 0.592$: insignificant

Part b: 0.5 point

Sample responses:

- Abnormally high losses to date would make chain ladder's estimate abnormally high since it is very reflective to losses to date. Additive model would not get impacted as much as C-L since the later years' future development for each period are independent and are not impacted by loss to date.
- C-L will assume future loss proportional to loss to date, so abnormally high loss to date will result in abnormally high loss for the next year. Additive model loss development will not be affected, as it is not linked to loss to date.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

- Additive model would not be influenced by the reported to date at an unusually high year. Chain Ladder model would have future losses in proportion to the unusually high losses to date, so the estimated future losses would be high.
- Chain ladder applies a multiplicative factor to losses to date, whereas the additive model adds a flat amount to losses known to date. The CL method would imply that since losses to date have been high, future losses will also be high, so the resulting estimate will be leveraged up. The additive model will have the same estimate of future losses regardless of level of losses reported to date.

EXAMINER'S REPORT

Candidates were expected to know the key assumptions of the chain ladder and additive reserving models, how to test these underlying assumptions, and be able to determine whether a particular model is or is not appropriate for the given situation.

Part a

Candidates were expected to calculate s_b , including all of the required components such as ϵ^2 and $(x_i - \bar{x})^2$. They were expected to recognize that because the slope was insignificant (since it was about the same as the standard error) that the LDF was statistically insignificant. Additionally, they were expected to know the underlying assumption of the chain ladder method.

Common mistakes included:

- Not stating the significance conclusion.
- Not stating the assumption.
- Not taking the square root of s_b^2 .

Part b

Candidates were expected to know that the future loss emergence for a year with abnormally high losses would be higher under a chain ladder methodology compared to an additive methodology. They were also expected to be able to explain why this was true.

Common mistakes included:

- Not providing any explanation as to why the conclusion was true.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 11

TOTAL POINT VALUE: 1.75

LEARNING OBJECTIVE(S): A7-A9

SAMPLE ANSWERS

Part a: 0.5 point

Sample 1

Definition: residuals do not share a common variance.

Issue: GLM assume residuals are iid (*independent identically distributed*). If not, we cannot sample the residuals from any part of the triangle.

Sample 2

Heteroscedasticity means the variance of residuals are not the same for the triangle

It violates the main assumption that residuals are independent and identically distributed.

Sample 3

Residuals are assumed to be independently and identically distributed. Therefore, variance of residuals should be constant. Heteroscedasticity is when variance of residuals it not constant. If the variance is not constant, some projected losses will be overestimated and some underestimated.

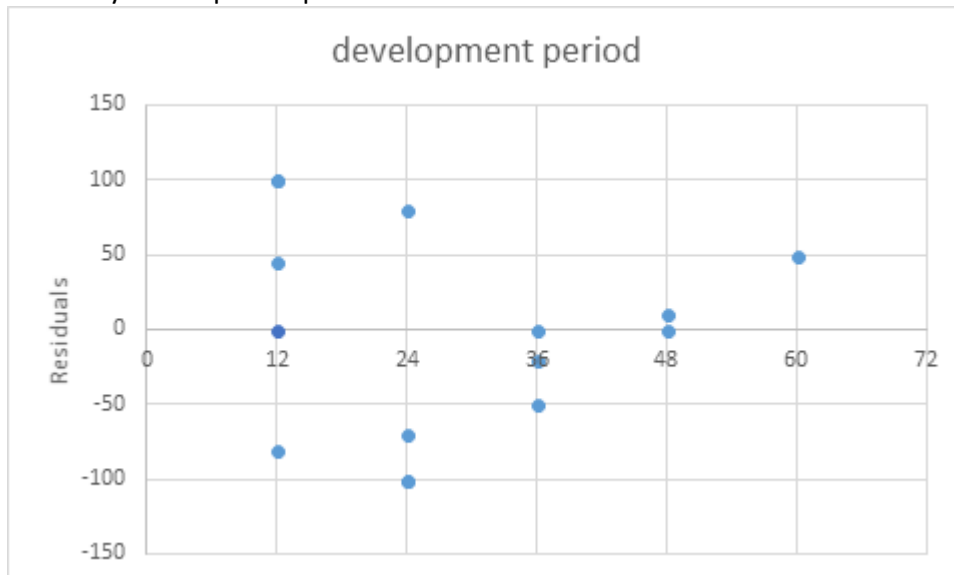
Sample 4

Heteroscedasticity occurs when the variance of residuals are not the same within the triangle. This results in bias when sampling losses in the GLM Bootstrap Model as residuals are not iid.

Part b: 0.75 point

Sample 1

Plot the residuals by development period.



From the plot above, heteroscedasticity exists since development periods 36-60 have a smaller spread/variance than 12-24.

Sample 2

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Mean of residuals @ 12mo = $(100 + 100 + -80 + 45 + 0) / 5 = 33$

Mean of residuals @ 24mo = -47.5

Mean of residuals @ 36mo = -23.33

Mean of residuals @ 48mo = 5

Variance of residuals @ 12mo = $(100^2 + 100^2 + -80^2 + 45^2 + 0^2) / 5 - 33^2 = 4,596$

Variance of residuals @ 24mo = 5,569

Variance of residuals @ 36mo = 422

Variance of residuals @ 48mo = 25

stdev @ 12mo = $\text{sqrt}(4,596) = 67.8$

stdev @ 24mo = 74.6

stdev @ 36mo = 20.5

stdev @ 48mo = 5

There appears to be development period heteroscedasticity as the standard deviation is significantly different between 12-24 and 36-48.

Sample 3

Heteroscedasticity seems to exist for development periods. Residuals in age 12 and 24 range from 100 to -80 and -100 to 80 respectively whereas the residuals in age 36 and 48 is more narrowly ranged.

Part c: 0.5 point

Sample 1

Stratified Sampling: Group residuals with similar variance and sample residuals from each group separately.

Sample 2

Using hetero-adjustment.

- i. Calculate the hetero adjustment factor after grouping residuals.
 $h_i = \text{Total standard deviation} / \text{standard deviation of group } i.$
- ii. Multiply h_i to standardized residuals in group $i.$
- iii. Sample the adjusted residuals as usual.
- iv. Un-adjust the hetero adjustment after sampling.

Sample 3

Use scale parameters.

Determine groups that show similar variance. Calculate the scale parameter to adjust the residuals. The scale parameter is $\text{sqrt}(\theta_{\text{total}}) / \text{sqrt}(\theta_i)$ where $\theta_i = \Sigma[\text{sqrt}(N/(N-P)) \times r]^2 / n_i.$ This would give hetero-adjusted residuals. After sampling, apply the reverse to get a residual for the development period.

EXAMINER'S REPORT

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Candidates were expected to understand what heteroscedasticity is, the issues associated with it, methods to identify the existence of it, and techniques to adjust residuals when it does exist.

Part a

Candidates were expected to know the definition of heteroscedasticity as well as issues associated with it. For the latter, multiple responses were accepted as long as the candidate showed understanding of the issue and it was tied to the GLM bootstrap model.

Common mistakes included:

- Stating "It is not homoscedastic" without showing further or sufficient understanding of the GLM bootstrap model framework.
- Not mentioning residuals when defining heteroscedasticity in the GLM bootstrap model framework (although credit was given if the candidate did not specifically mention residuals but did imply it in other parts of the question).
- Stating that the incremental losses are iid instead of residuals being iid.
- Talking about non-constant distribution instead of non-constant variance. Distributions can be different but still have the same variance.

Part b

Candidates were expected to know methods to identify the existence of heteroscedasticity. Candidates needed to state that the variance/spread/range of the later development periods were smaller than the early periods and provide proof backing up the argument.

Common mistakes included:

- Only stating that variance is different without providing support.
- Evaluating for heteroscedasticity along accident year or calendar year, since the question specifically asked for development year.
- Forgetting to tie the change in variance of residuals back to whether or not heteroscedasticity exists.
- Stating that there are patterns or trends in the residuals. Heteroscedasticity requires non-constant variances. If patterns or trends exist, that does not necessarily imply that the variance is changing.
- Including/using the sum of squared residuals. Although similar to variance, they are not the same thing.

Part c

Candidates were expected to know techniques to adjust residuals when heteroscedasticity does exist. The candidate was only expected to know the steps and theory behind these techniques, not the actual formulas and equations.

Common mistakes included:

- Only stating "split into groups" or "group like residuals" when talking about stratified sampling. Grouping development periods into groups with similar variances was required

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

for full credit. If the candidate had already implied groups with similar variances in part b, full credit was given for part c.

- Not removing the adjustment factor after sampling. If the adjustment factor is not removed, the sampled residuals are biased.

SPRING 2018 EXAM 7, QUESTION 12

TOTAL POINT VALUE: 2

LEARNING OBJECTIVE(S): A5

SAMPLE ANSWERS

Part a: 1 point

Sample 1

- Specification error: The risk that the model cannot fully capture the complexities of the insurance process.
- Parameter selection error: The risk that the selected parameters & their trends do not match the true parameters & their trends.

Sample 2

- Specification error: Catastrophes are complex to model and it will be difficult to accurately capture the intricacies in an internal model.
- Parameter estimation error: We're looking at a tiny amount of data for hurricanes; 5 years is not enough time to get an accurate frequency view. Severity will also be difficult to estimate as there is only 1 event.

Sample 3

- Specification error: the error resulting from the insurance process being too complex for any model to fully capture. This is applicable because hurricane & other CAT exposures are very volatile & complex.
- Parameter selection error: the error from being unable to adequately measure predictors of claim cost outcomes & trends in these predictors. Insurer has very limited information to fit parameters from (only 1 hurricane & only going to test 10 years of data).

Sample 4

- Specification risk: only one model is used in generating results. You could improve it by using multiple models to capture strengths of various models.
- Parameter error: The selected parameters could be based on historical data that has changed in recent years. Could cause an understatement in future losses.

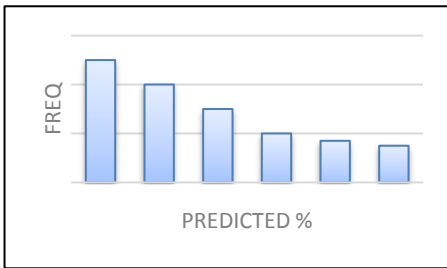
Sample 5

- Specification error: did we use multiple models to get to our answer or just one? Best practice would be to consider several models.
- Parameter selection: Are our predictors generated close to the best predictors? Have we identified the best predictors? Data seems thin so I'm doubtful.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

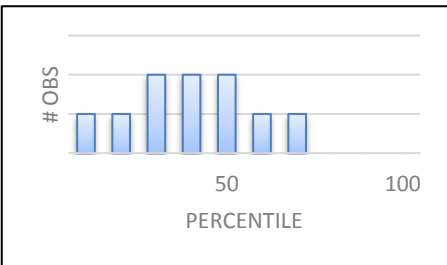
Part b: 1 point

Sample 1



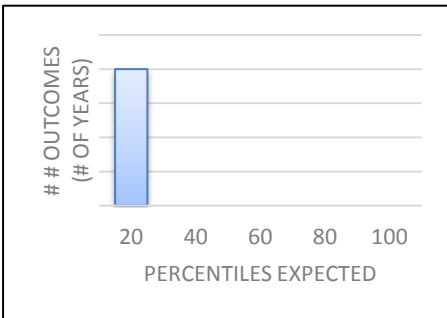
Model overestimates losses, as it expects cats in a couple years at least. With no cats, losses come in lower than expected, so our model is predicting too high (biased high).

Sample 2



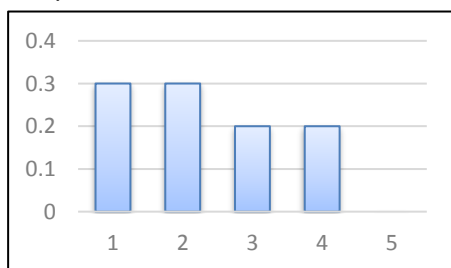
The model results would appear biased high. Most actual outcomes would be in low percentiles due to absence of cat losses.

Sample 3



Compared to a period where hurricane losses made up almost 80% of losses, non-hurricane years will always be much lower than the model predictions.

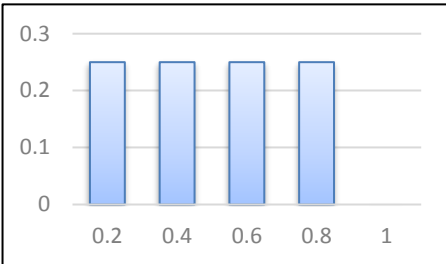
Sample 4



I expect most results would fall in the lower expected percentiles because there were no hurricanes.

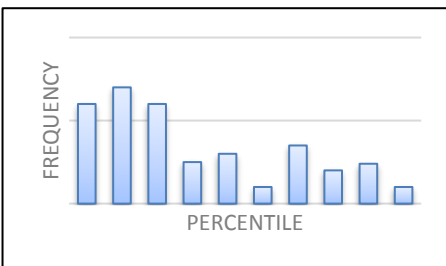
EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 5



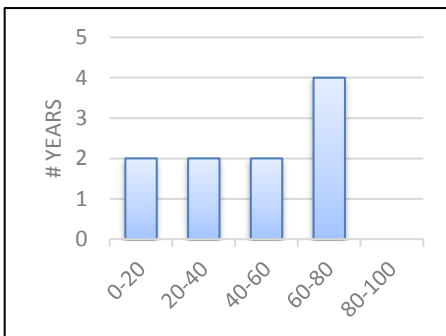
Based on historical data, hurricane happens 1 in 5 years. Since no hurricane in the next ten years, 0 in (80%-100%) and 0.25 in (0,20%), (20%,40%),...

Sample 6



Model will be biased high (due to no hurricane losses) and is likely overfit. There will be more volatility due to overfitting. (Shape at higher percentiles due to overfitting).

Sample 7



Hurricanes are rare, and my guess is that including the hurricane losses without adjustment in only 5 years of data has caused the model to be overfit and biased high. Since hurricanes occurred in 1 out of 5 years in the data, all years in the test period will be low 80th percentile of distribution. The other losses will probably be too low because there is a positive severity trend that wasn't adjusted for in the data, so below 80th percentile, model is biased low, so 60-80 will be higher.

EXAMINER'S REPORT

Candidates were expected to describe the systemic risks present in the construction of a stochastic reserve model and evaluate their potential impact on the model prediction by means of a histogram showing actual future outcomes versus the percentiles implied by the stochastic reserve model.

Part a

Candidates were expected to identify and briefly describe the two applicable internal systemic risks, Specification Error & Parameter Selection Error.

Common mistakes included:

- Other risks/errors named (e.g. model risk, estimation error, projection error, data quality risk, event risk, process risk).
- Vague or incomplete descriptions of Specification Error (e.g. risk of not selecting the correct/right model).

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

- Risk of using one model.
- Mentioning Data Error even though the question specifically stated to assume no data errors.

Part b

Candidates were expected to draw and explain a hypothetical histogram from the results of a retrospective test conducted on this stochastic reserve model using the next 10 years of data. Given that the model was fit with a high correlation coefficient on an experience period including a hurricane event which generated a high volume of losses, and the next 10 years gave rise to no hurricane events, that would imply a high bias, or possibly heavy tail, manifesting itself in a histogram that has more mass in the lower percentiles anticipated by the model. There was wide latitude in the types of histograms and associated shapes accepted, as long as the histogram considered the data and implications of actual vs. expected emergence with an appropriate explanation.

Common mistakes included:

- Providing a chart of the estimated losses ordered by accident year, rather than a histogram with expected loss percentiles or buckets.
- Providing a chart that is not clearly a histogram.
- Providing histograms that show expected losses, but do not consider the retrospective test of the model's prediction.
- Not recognizing the bias that the hurricane loss may cause in the model predictions of future non-hurricane loss years.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 13

TOTAL POINT VALUE: 2.75

LEARNING OBJECTIVE(S): A7, A8

SAMPLE ANSWERS

Part a: 0.5 point

Sample responses:

- It cannot handle negative incremental values
- It cannot handle incomplete triangle
- Must be solved at each iteration making it time consuming
- GLM bootstrap requires computation of parameters at every step which can be computationally intensive
- Heteroscedasticity – when residuals don't have similar variance and they are not adjusted, GLM won't be appropriate or produce unstable results
- Make iid assumption in bootstrapping which won't work if non-constant variance across triangle
- Not easily explained to non-technical audience
- Sometimes the sum of residuals does not equal zero
- When the entire column sum of incremental losses is negative the GLM again will fail
- Can't incorporate CY trend
- ODP bootstrap GLM uses large number of parameters and may overfit data
- Needs adjustment for growth/shrinkage in book
- It cannot incorporate the deductibles and the policy limits as variable
- May underestimate variability if underlying triangle is limited & does not exhibit enough variability, as is the case with a small triangle

Part b: 1.25 points

Sample 1

Case Reserve Triangle

AY	12	24	36
2015	15 × 3=45	10	0
2016	75	30	
2017	90		

↑ ↑
 15K per 10K per
 open claim open claim

Open count × case reserve for each maturity

Cum Paid Triangle

AY	12	24	36
2015	135	220	235
2016	125	150	
2017	135		

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Incremental Paid Triangle

AY	12	24	36
2015	135	85	15
2016	125	25	
2017	135		

Take log of each cell

Log of Incremental Paid

AY	12	24	36
2015	4.91	4.44	2.71
2016	4.83	3.22	
2017	4.91		

Sample 2

Reported – # Open × Reserve = Paid

Incremental Paid

AY	12	24	36
2015	135,000	85,000	15,000
2016	125,000	25,000	
2017	135,000		

AY	12	24	36
2015	11.813	11.350	9.616 ← Log(15,000)
2016	11.736	10.127	
2017	11.813		

Part c: 1 point

Sample 1

$\ln q_{11} = \alpha_1 = 5.04$

$\ln q_{21} = \alpha_2 = 4.66$

$\ln q_{31} = \alpha_3 = 4.91$

$\ln q_{12} = \alpha_1 + \beta_2 = 4.18$

$\ln q_{22} = \alpha_1 + \beta_2 = 3.8$

$\ln q_{13} = \alpha_1 + \beta_2 + \beta_3 = 2.71$

$q_{11} = 154$

$q_{21} = 106$

$q_{31} = 136$

$q_{12} = 65$

$q_{22} = 45$

$q_{13} = 15$

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Fitted incremental paid loss			
AY	12	24	36
2015	154	65	15
2016	106	45	
2017	136		

Sample 2

	e^{α_1}	$e^{\alpha_1 + \beta_2}$	$e^{\alpha_1 + \beta_2 + \beta_3}$
AY	12	24	36
2015	154,500	65,400	15,000
2016	106,600	44,700	
2017	135,600		

EXAMINER'S REPORT

Candidates were expected to know background on the log-link GLM bootstrap modeling framework, including knowing how to construct a log-link triangle, and how to calculate fitted incremental paid results.

Part a

Candidates were expected to know practical limitations of log-link GLM bootstrap modeling framework.

Common mistakes included:

- Only providing one response.

Part b

Candidates were expected to construct a log-link triangle that can be used to solve for the GLM parameters.

Common mistakes included:

- Basing the calculation on incremental reported losses.

Part c

Candidates were expected to be able to calculate the fitted incremental paid results of the GLM model using given parameters.

Common mistakes included:

- Using an incorrect equation.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 14	
TOTAL POINT VALUE: 1.25	LEARNING OBJECTIVE(S): A11, A12
SAMPLE ANSWERS	
Part a: 0.5 point	
<p><u>Sample 1</u></p> <ul style="list-style-type: none"> • The mode is likely to be low due to most claims being small. This would increase the report lag to the reinsurer. • Loss (and ALAE) will be consistently under reserved for serious claims. <p><u>Sample 2</u></p> <ul style="list-style-type: none"> • Reinsurers generally do not hear about a claim until it pierces the reinsurance layer, so if mode is reserved, then report lag for this potentially serious claim will likely be longer. • Reinsurance claims tend to have persistent upward development, so reserving to the mode is likely not reasonable for an initial reserve. 	
Part b: 0.75 point	
<p><u>Sample 1</u></p> <ul style="list-style-type: none"> • Industry statistics are not very useful • Difficult to aggregate data to increase credibility • Data coding and IT issues <p><u>Sample 2</u></p> <ul style="list-style-type: none"> • Traditional reserve methods require homogenous groups of data. You need alternate methods since credibility becomes an issue when splitting reinsurance data. • Coding and IT systems can get backed up and have processing errors due to many different requirements/handling of heterogeneous data. • Industry data is not very useful due to heterogeneity of reinsurer's data. 	
EXAMINER'S REPORT	
Candidates were expected to know basic background on reinsurance reserving, including specific challenges.	
Part a	
Candidates were expected to cite the two challenges that arose for a reinsurer due to modal reserving by the primary insurer.	
Common mistakes included:	
<ul style="list-style-type: none"> • Stating other problems with reinsurance reserving unrelated to modal reserving (e.g. high reserve to surplus ratio, difficulty convincing management to book the high reserve amount). • Stating the same challenge (as described by Patrik) twice (e.g. persistent upward development <i>and</i> ALAE under reserved). 	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part b

Candidates were expected to cite the three challenges in reinsurance reserving related to the heterogeneity of exposures.

Common mistakes included:

- Restating the question as an answer (i.e. “having heterogeneous exposures makes reinsurance reserving difficult”).
- Simply defining heterogeneity without explaining why it was challenging for reinsurance reserving (e.g. simply stating that the reporting patterns/mix of business/etc. were different amongst heterogeneous exposures without explaining the challenge).
- Stating other problems with reinsurance reserving unrelated to heterogeneity (e.g. high reserve to surplus ratio).
- Stating the same challenge (as described by Patrik) twice (e.g. industry data is hard to find *and* Schedule P is too highly aggregated).

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 15	
TOTAL POINT VALUE: 2.25	LEARNING OBJECTIVE(S): A8
SAMPLE ANSWERS	
Part a: 0.75 point	
<u>Sample 1</u>	
<ul style="list-style-type: none">i. The claim reporting pattern has changed. With more claims staff, adjusters are able to spend more time setting initial case reserves than in prior years. The latest year case reserves are probably more sufficient so we should expect less development in the future.ii. We should incorporate expert opinion by selecting a LDF for the latest year that reflects the expected change in loss reporting (a lower LDF).iii. If no adjustments are made then IBNR will be too high.	
<u>Sample 2</u>	
<ul style="list-style-type: none">i. Speedup in claim payment for these years.ii. Reduce the expected LDF.iii. IBNR would be overstated.	
Part b: 0.75 point	
<u>Sample 1</u>	
<ul style="list-style-type: none">i. Rapid growth in CAT prone areas may or may not appear in the loss data. If no catastrophes have occurred, then the loss data will be unaffected but the risk will still exist.ii. We should incorporate expert opinion by using a CAT model to have a better understanding of the loss experience and development should be analyzed separately from non-CAT losses. The CAT model results can be used to set reserves for events that occurred but we are unsure still of the total loss.iii. If no adjustments are made, IBNR would be inappropriate as non-CAT and CAT losses can have very different reporting patterns, or risk from CATs won't be reflected in IBNR.	
<u>Sample 2</u>	
<ul style="list-style-type: none">i. Growth in premium, volatility in losses in recent years.ii. Calculate IBNR for property cat losses separately.iii. Year to year volatility in IBNR.	
Part c: 0.75 point	
<u>Sample 1</u>	
<ul style="list-style-type: none">i. The loss data will be impacted by showing negative development from one large claim. This could be leveraging a development pattern.ii. This claim should be removed from the data since it is an anomaly. We don't want it to affect the age-to-age factors or IBNR estimates.iii. If no adjustments are made and the claim is left in the data, the selected age-to-age factor will be lower than it should be and IBNR will be too low.	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 2

- i. A negative development b/w the two ages that the settlement happens.
- ii. Remove this outlier from the loss data.
- iii. IBNR would be understated.

EXAMINER'S REPORT

Candidates were expected to be able to identify the impact of the situations on the loss data and on the IBNR. They were also expected to propose an adjustment using expert opinion that corrected the impact on IBNR.

Part a

Candidates were expected to know that more claim staff means that reserves will show a shorter development pattern, and also the impact of a shorter development pattern on the IBNR calculated with the chain-ladder method.

Common mistakes included:

- Identifying claim adjusters as the expert opinion, since they are not considered experts on the same level as reserving actuaries with respect to IBNR and would not be making adjustments to the data.

Part b

Candidates were expected to acknowledge that including catastrophe losses with attritional losses for reserving purposes would create distortions in the resulting IBNR.

Common mistakes included:

- Stating that catastrophe losses should be removed from the triangle, but without mentioning that they should be either added back through a cat loading or analyzed separately with a cat model.

Part c

Candidates were expected to identify that the downward development would have an impact on the age-to-age factor at the time of settlement, which in turn would create understated IBNR if the downward development is identified as an outlier.

Common mistakes included:

- Stating that no adjustment is necessary because the claim is closed, thus will not further develop and that this is included in the calculated IBNR. This ignores the impact on the LDF and the expected development of other claims.
- Proposing as an adjustment to lower the case reserve for that claim, assuming that the downward development was not already reflected in the triangle. This is not a valid assumption.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 16

TOTAL POINT VALUE: 3

LEARNING OBJECTIVE(S): A13

SAMPLE ANSWERS

Part a: 2.5 points

Sample 1

AY	Reported Loss	Used-Up Premium	SB IBNR	CL IBNR
2014	3,000	$6,000 \times 0.7 = 4,200$	$6,000 \times 0.8411 \times (1-0.7) = 1,513.98$	$3000 / 0.7 - 3,000 = 1,285.71$
2015	2,000	$7,000 \times 0.5 = 3,500$	$7,000 \times 0.8411 \times (1-0.5) = 2,943.85$	$2000 / 0.5 - 2,000 = 2,000$
2016	4,000	$10,000 \times 0.3 = 3,000$	$10,000 \times 0.8411 \times (1-0.3) = 5,887.7$	$4000 / 0.3 - 4,000 = 933.33$
Total	9,000	10,700	ELR = 9,000 / 10,700 = 0.8411	

AY	Z	Cred IBNR	Expected Emergence
2014	$0.5 \times 0.7 = 0.35$	$0.35 \times 1,285.71 + (1 - 0.35) \times 1,513.98 = 1,434.09$	$1,434.09 \times (0.75 - 0.7) / (1 - 0.7) = 239.02$
2015	$0.5 \times 0.5 = 0.25$	$0.25 \times 2000 + (1 - 0.25) \times 2,943.85 = 2,707.89$	$2,707.89 \times (0.55 - 0.5) / (1 - 0.5) = 270.79$
2016	$0.5 \times 0.3 = 0.15$	$0.15 \times 9,333.33 + (1 - 0.15) \times 5,887.7 = 6,404.54$	$6,404.54 \times (0.35 - 0.3) / (1 - 0.3) = 457.47$

AY	Actual Emergence	Difference
2014	$3,500 - 3,000 = 500$	$500 - 239.02 = 260.98$
2015	$2,200 - 2,000 = 200$	$200 - 270.79 = -70.79$
2016	$4,800 - 4,000 = 800$	$800 - 457.47 = 342.53$
Total		532.72

Actual emergence higher than expected emergence

Sample 2

AY	(1) Adj Prem	(2) Report Lag	(3)=(1)×(2) Used-up premium	(4) Reported Losses	(5) Reported Lag at 6/30/2017
2014	6,000	0.7	4,200	3,000	0.75
2015	7,000	0.5	3,500	2,000	0.55
2016	10,000	0.3	3,000	4,000	0.35
			10,700	9,000	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

$(6)=(5)-(2)$	$(7)=(6)\times(1)\times ELR$	$(8)=(4)/(2)\times(6)$	$(9)=(2)\times 0.5$	$(10)=(9)\times(8)$ $+ [1-(9)]\times(7)$
Lag Difference	Expected Rep Loss using CC	Expected Rep Loss using CL	z	Expected IncLoss Cred
0.05	252	214	0.35	239
0.05	294	200	0.25	271
0.05	421	667	0.15	458
				968

Expected Loss Emerged = 968
 Actual Loss Emerged = 10,500 – 9,000 = 1,500
 Expected – Actual = -532

Part b: 0.5 point

Sample responses:

- Our estimate was too low so perhaps our initial IBNR was too low.
- Maybe it's just random chance.
- The company begins a new monitoring program that increases how fast claims are reported to the company.
- The company increases the average case reserves due to a management change.
- The claim report lag is too short.
- We may have had a large loss.
- Case load effect: losses are being reported faster than usual because there are fewer claims per claims staff member.
- Legislative change: law might have shortened the statute of limitations on a claim so claimants are reporting faster than before the lag was set.

EXAMINER'S REPORT

Candidates were expected to know how to calculate expected loss emergence and why the actual emergence could be different than expected.

Part a

Candidates were expected to know how to calculate expected loss emergence using both Chain-Ladder and Standard-Bühlmann methods, and how to credibility weight the two methods.

Common mistakes included:

- Not multiplying the credibility factor by the lags.
- Using the wrong evaluation date values.
- Incorrectly applying credibility to CL and SB.

Part b

Candidates were expected to understand why the actual emergence could be different than expected.

Common mistakes included:

- Providing only one reason.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

- Simply stating that losses developed more than expected.
- Claiming that the credibility factor was too low without justification. If part a. is done correctly, the actual is significantly higher than expected for either of the methods.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 17	
TOTAL POINT VALUE: 2.25	LEARNING OBJECTIVE(S): A14
SAMPLE ANSWERS	
Part a: 1.25 points	
<u>Sample 1</u>	
$CPDLD1 = \frac{1.75(0.785) + 0.7(0.1) + 0.55(0.07) + 0.45(0.04) + 0(0.005)}{(0.785 + 0.1 + 0.07 + 0.04 + 0.005)} = 1.5$	
<u>Expected Future Loss Emergence:</u>	
14: 226,500 – 202,000 = 24,500	
15: 225,000 – 180,500 = 44,500	
16-17: (200,000 + 80,000) – (0+0) = 280,000	
<u>Expected Future Premium (= Expected Future Loss Emergence × CPDLD)</u>	
14: 24,500 × 0.49 = 12,005	
15: 44,500 × 0.59 = 26,255	
16-17: 280,000 × 1.5 = 420,000	
<u>Estimated Total Premium (= Expected Future Premium + Premiums Booked from Prior Adjustment)</u>	
14: 12,005 + 335,000 = 347,005	
15: 26,255 + 330,000 = 356,225	
16-17: 420,000 – 0 = 420,000	
Sum of Estimated Total Premium for All Years: 458,260	
<u>Premium Asset by Year (= Estimated Total Premium - Premiums Booked as of December 2017)</u>	
14: 347,005 – 337,000 = 10,005	
15: 356,225 – 335,000 = 21,255	
16-17: 420,000 – 425,000 = -5,000	
Total Premium Asset = 10,005 + 21,255 – 5,000 = 26,260	
<u>Sample 2</u>	
$CPDLD1 = \frac{1.75 \times (0.785) + 0.7 \times (0.1) + 0.55 \times (0.07) + 0.45 \times (0.04) + 0 \times (0.005)}{(0.785 + 0.1 + 0.07 + 0.04 + 0.005)}$ $= 1.50025$	
<u>Expected Future Premium</u>	
14: 226,500 – 202,000 = 24,500 × 0.49 = 12,005	
15: 225,000 – 180,500 = 44,500 × 0.59 = 26,255	
16-17: 280,000 – 0 = 280,000 × 1.50025 = 420,070	
Total = 12,005 + 26,255 + 420,070 = 458,330	
Premium at Prior Retro = 330,000 + 335,000 = 665,000	
Premium Booked = 337,000 + 335,000 + 210,000 + 215,000 = 1,097,000	
Total = 458,330 + 665,000 – 1,097,000 = 26,330	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part b: 1 point

Sample responses:

- More losses are piercing the individual claim caps/per-accident limit or per-occurrence limits as development continues, which leads to more losses than premium to the insurer, decreasing the PDL.
- The first period includes basic premium that contemplates certain expenses to be paid once. The remaining PDL are calculated on an incremental loss emergence basis – so they will decrease as time increases.
- More losses have already been capped in the numerator of the loss capping ratio, but the denominator is uncapped and keeps increasing with a static/ low increasing numerator and a steadily increasing denominator, the PDL ratios decrease.
- As the period increases, the retro premium is more likely to be impacted by the maximum, so while losses would still be developing, there would be no more premium development since it is at the maximum premium.
- Over time the retro rating parameters of the book of business shifted (e.g. per-accident limit decreases) and PDL ratios decline.

EXAMINER'S REPORT

Candidates were expected to understand the formula for deriving a retrospective premium asset. Candidates were also expected to demonstrate an understanding of how and why retrospective premium develops relative to losses over time.

Part a

Candidates were expected to calculate the correct premium asset.

Common mistakes included:

- Applying the wrong CPDL ratios to the policy year reserves.
- Applying the CPDL ratios to ultimate losses instead of loss reserves.
- Not including the calculation of 2017, and giving a reason for not including as "Policy Year is too recent" or "the amount was less than \$0".

Part b

Candidates were expected to describe why the PDL ratios decrease over time.

Common mistakes included:

- Providing incomplete responses (e.g. listing versus describing, or only providing a single response).
- Stating the PDL ratio decreases because there is less loss development over time.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 18	
TOTAL POINT VALUE: 3.5	LEARNING OBJECTIVE(S): B3
SAMPLE ANSWERS	
Part a: 1 point	
<p><u>Sample 1</u> Cost of Capital (k) = $(0.03 - 0.01) + 1.5 \times 0.06 = 0.11$</p> <p>Growth Rate (g) = $(1 - \text{dividend payout rate}) \times \text{projected ROE} = (1 - 0.5) \times 0.12 = 0.06$</p> <p>Price/BV = $1 + \frac{\text{ROE} - k}{k - g}$ = 1.2</p> <p><u>Sample 2</u> Risk-free rate r(f) = $3\% - 2\% = 1\%$ Risk Premium = $E[r(m) - r(f)] = 6\%$ (given) Cost of Capital k = $r(f) + \text{beta} \times \text{Risk Premium} = 1\% + 1.5 \times 6\% = 11\%$ Growth rate g = $\text{Plowback ratio} \times \text{ROE} = (1 - 50\%) \times 12\% = 6\%$ P-BV = $1 + (\text{ROE} - k) / (k - g) = 1 + (12\% - 11\%) / (11\% - 6\%) = 1.2$</p>	
Part b: 0.5 point	
<p><u>Sample 1</u> Price/BV = $1 + [(ROE - k)/(k - g)] \times (1 - [(1 + g)/(1 + k)]^3)$ = $1 + [(0.12 - 0.11)/(0.11 - 0.06)] \times (1 - [(1 + 0.06)/(1 + 0.11)]^3)$ = 1.026</p> <p><u>Sample 2</u> Price/BV = $1 + \frac{(0.12 - 0.11)}{1.11} + \frac{(0.12 - 0.11) \times 0.66}{1.11^2} + \frac{(0.12 - 0.11) \times 0.33}{1.11^3}$ = 1.017</p>	
Part c: 0.5 point	
<p><u>Sample 1</u> P-BV in part (b) is more reasonable since it assumes abnormal returns decline to zero as competitors enter the market to capture the excess returns. This is more reasonable.</p> <p><u>Sample 2</u> The return horizon of 3 years of excess returns (in the part (b)) is more reasonable because over time competitors will enter the market and reduce profits.</p>	
Part d: 0.5 point	
<p><u>Sample 1</u> Transaction multiples are based on complex negotiations between sophisticated parties. This is more meaningful than the market multiples in (a, b) that are more volatile.</p>	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 2

Since the transaction multiples are obtained after complex negotiations between sophisticated parties, these multiples are more meaningful and less prone to random fluctuations.

Part e: 1 point

Sample responses:

- Control premiums – firms may overpay to acquire the target firm and control its operations.
- M&A overpricing – mergers and acquisitions lead to increase in the target firm's value, implying that the acquiring firm is overpaying.
- IPOs underpricing – IPOs tend to be underpriced, based on studies of historical transactions.
- Different economic environment – key economic variables contained in historic multiples may reflect an economic environment that is not representative of the current environment.

EXAMINER'S REPORT

Candidates were expected to calculate Price to Book Value (P-BV), understand the assumptions underlying the calculations, and know the advantages and disadvantages of market multiples vs. transaction multiples (IPOs and M&A).

Part a

Candidates were expected to calculate the Price to Book Value multiple given the set of financial information and assuming the abnormal earnings continued in perpetuity.

Common mistakes included:

- Miscalculating the cost of capital. Some candidates used the yield on long-term T-bonds (3.0%) as the risk-free rate in the CAPM equation, while others subtracted risk-free rate from the expected equity risk premium value that was given.
- Assuming that growth rate is 0%
- Using the wrong formula to calculate P-BV value

Part b

Candidates were expected to calculate the Price to Book Value assuming excess returns decline in 3 years.

Common mistakes included:

- Attempting to calculate a tail factor after 3 years. If the excess returns decline to zero, then there is no tail factor.
- Setting $k = \text{ROE}$ immediately in the first year, and calculating a resulting P-BV of 1.0.

Part c

Candidates were expected to identify that the P-BV in part b. would be a more reasonable indicator and explain that a company with abnormal earnings would see increased competition, making it unreasonable to earn abnormal earning in perpetuity.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

No credit was given to responses that identified the value in part a. as the most reasonable, because the company has a beta greater than 1.0 and, as a result, is taking risk and expecting to grow. A company with increased volatility cannot ensure positive abnormal earning in perpetuity.

Common mistakes included:

- Identifying that abnormal returns in perpetuity were unreasonable but failing to explain why.

Part d

Candidates were expected to explain that IPOs and M&A involve complex negotiations and sophisticated parties, making them more meaningful than market multiples.

Common mistakes included:

- Stating that transaction multiples are based on additional sources of data, as compared to the market multiples.

Part e

Candidates were expected to describe two reasons why multiples based on financial data may be better than multiples based on IPOs and M&A.

Common mistakes included:

- Reversing the fact that IPOs are usually underpriced and M&As are usually overpriced.
- Stating that IPOs are generally overpriced because investors are often overly optimistic of newly public companies.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 19	
TOTAL POINT VALUE: 2.5	LEARNING OBJECTIVES: B2, B3
SAMPLE ANSWERS	
Part a: 1.5 points	
<p><u>Sample 1</u> Discount Rate = $4.0\% + 0.80 \times 5.0\% = 8.0\%$</p> <p>2018 AE = $22,000 - 189,200 \times 8.0\% = 6,864$ 2019 AE = $28,000 - 190,300 \times 8.0\% = 12,776$ 2020 AE = $34,000 - 191,700 \times 8.0\% = 18,664$</p> <p>2021 AE = $18,664 \times 2/3 = 12,443$ 2022 AE = $18,664 \times 1/3 = 6,221$</p> <p>Equity = $189,200 + 6,864/1.08 + 12,776/1.08^2 + 18,664/1.08^3 + 12,443/1.08^4 + 6,221/1.08^5$ = 234,705</p> <p>Difference = $234,705 - 276,716 = -42,011$</p>	
<p><u>Sample 2</u> $4.0\% + 0.80 \times 5.0\% = 8.0\%$</p> <p>2018 ROE = $22 / 189.2 = 11.6\%$ 2019 ROE = $28 / 190.3 = 14.7\%$ 2020 ROE = $34 / 191.7 = 17.7\%$</p> <p>$(11.6\% - 8.0\%) \times 189.2 = 6.8$ $(14.7\% - 8.0\%) \times 190.3 = 12.75$ $(17.7\% - 8.0\%) \times 191.7 = 18.6$</p> <p>$18.6 \times 2/3 = 12.4$ $18.6 \times 1/3 = 6.2$</p> <p>$189.2 + 6.8/1.08 + 12.75/1.08^2 + 18.6/1.08^3 + 12.4/1.08^4 + 6.2/1.08^5 = 234.5$</p> <p>$234.5 - 276.7 = -42.2$</p>	
<p><u>Sample 3</u> $4.0\% + 0.80 \times 5.0\% = 8.0\%$</p> <p>2020 AE = $34,000 - 191,700 \times 8.0\% = 18,664$</p> <p><u>When AE decreases linearly to zero in 2023:</u> 2021 AE = $18,664 \times 2/3 = 12,443$ 2022 AE = $18,664 \times 1/3 = 6,221$</p>	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

$$\text{Present Value of AE} = 12,443/1.08^4 + 6,221/1.08^5 = 13,380$$

When AE decreases linearly to zero in 2031:

$$2021 \text{ AE} = 18,664 \times 10/11 = 16,967$$

$$2022 \text{ AE} = 18,664 \times 9/11 = 15,271$$

$$2023 \text{ AE} = 18,664 \times 8/11 = 13,574$$

$$2024 \text{ AE} = 18,664 \times 7/11 = 11,877$$

$$2025 \text{ AE} = 18,664 \times 6/11 = 10,180$$

$$2026 \text{ AE} = 18,664 \times 5/11 = 8,484$$

$$2027 \text{ AE} = 18,664 \times 4/11 = 6,787$$

$$2028 \text{ AE} = 18,664 \times 3/11 = 5,090$$

$$2029 \text{ AE} = 18,664 \times 2/11 = 3,393$$

$$2030 \text{ AE} = 18,664 \times 1/11 = 1,697$$

$$\text{PV of AE} = 16,967/1.08^4 + \dots + 1,697/1.08^{13} = 55,391$$

$$\text{Difference} = 13,380 - 55,391 = -42,011$$

Sample 4

$$4.0\% + 0.80 \times 5.0\% = 8.0\%$$

$$2020 \text{ AE} = 34,000 - 191,700 \times 8.0\% = 18,664$$

PV of difference in AE's:

$$18,664 \times [$$

$$(2/3 - 10/11) / 1.08^4 +$$

$$(1/3 - 9/11) / 1.08^5 +$$

$$(-8/11) / 1.08^6 +$$

$$\dots +$$

$$(-1/11) / 1.08^{13}]$$

$$= -42,011$$

Part b: 1 point

Sample 1

Under 2-year assumption:

$$P/BV = 234,705 / 189,200 = 1.24$$

$$P/E = 234,705 / 22,000 = 10.67$$

Under 10-year assumption:

$$P/BV = 276,716 / 189,200 = 1.46$$

$$P/E = 276,716 / 22,000 = 12.58$$

The 10-year ratios are more in line with peer companies for both P/BV and P/E. Therefore, 10-year is more reasonable.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 2

Average of peer P/E = $(12.5 + 13.4 + 12.5 + 12.9) / 4 = 12.825$

Estimated equity = $12.825 \times 22,000 = 282,150$

Average of peer P/BV = $(1.29 + 1.59 + 1.37 + 1.76) / 4 = 1.5025$

Estimated Equity = $1.5025 \times 189,200 = 284,273$

These equity values are closer to the equity under the 10-year assumption (276,716) so 10 years is more reasonable.

Sample 3

Peer equities based on P/E:

Smallest Peer P/E: $12.5 \times 22,000 = 275,000$

Largest Peer P/E: $13.4 \times 22,000 = 294,800$

Peer equities based on P/BV:

Smallest Peer P/BV: $1.29 \times 189,200 = 244,068$

Largest Peer P/BV: $1.76 \times 189,200 = 332,992$

The equity under the 2-year assumption is outside these ranges while the equity under the 10-year assumption is within these ranges, so 10-years is more reasonable.

EXAMINER'S REPORT

Candidates were expected to demonstrate understanding of expected abnormal earnings, to use the estimated abnormal earnings to estimate the equity value of a P&C insurer, and to evaluate and justify the reasonableness of different assumptions by using peer company financial ratios.

Part a

Candidates were expected to calculate abnormal earnings and use them to estimate the equity value of a company. There were multiple reasonable approaches to receive full credit when calculating the difference. The most common were:

- Calculate the equity under the "zero-in-2023" scenario, and then take the difference from the equity value provided in the question. This approach usually involved the least amount of calculation for the candidate.
- Calculate the abnormal earnings under both scenarios and then take the difference between the equity values, the present values of all abnormal earnings, or the present values of just the tail values of the two scenarios. All three options would give the same result because the starting book value and the first three abnormal earning values are the same under both scenarios.

Common mistakes included:

- Using an incorrect formula to calculate the abnormal earnings from the net incomes, starting GAAP equities, and the calculated discount rate.
- Using required returns instead of abnormal earnings.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

- Incorrectly linearly decreasing the abnormal earnings under either scenario. The most common example of this error was to use $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ of the 2020 abnormal earnings over the next three years, respectively, instead of using $\frac{2}{3}$ and $\frac{1}{3}$ of the 2020 abnormal earnings over the next two years. Under the first set of ratios, the abnormal earnings do not decrease linearly to zero in 2023, as the question states.
- Comparing the undiscounted tail value of the two scenarios.
- Forgetting to take the difference between the equity values of the two scenarios.
- While technically not a mistake, some spent time calculating abnormal earnings that were already provided in the question.

Part b

Candidates were expected to use the peer company ratios provided in the question to determine which of the two different scenarios was more reasonable.

Common mistakes included:

- Using the equity value of 276,716 as the denominator instead of the starting GAAP equity of 189,200.
- Using the book values from 2019 or 2020 instead of 2018.
- Claiming that two years is more reasonable simply because it is fewer than ten years, with no justification for why abnormal earnings are not expected to last forever.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 20

TOTAL POINT VALUE: 1.5

LEARNING OBJECTIVE: C2

SAMPLE ANSWERS

Sample 1

	<u>Current</u>	<u>Option 1</u>	<u>Option 2</u>
Capital held	2,000	1,500	1,100
ΔCapital	-	-500	-900
Net Prem =Direct-Ceded	1,000	780	670
Net Profit =(Prem)(1-CR)	200	132.6	93.8
ΔProfit	-	-67.4	-106.2
Marginal ROE = ΔProfit / ΔCapital		13.48%	11.8%

Sample 2

$$\text{Marginal ROE} = \frac{\text{Net Cost of Reinsurance}}{\text{Capital Freed}}$$

$$\text{Capital Freed}_1 = 2,000 - 1,500 = 500$$

$$\text{Capital Freed}_2 = 2,000 - 1,100 = 900$$

$$\text{Net Cost}_1 = \text{Ceded Prem} - \text{Recoveries} = 220 - 152.6 = 67.4$$

$$CR_1 = 0.83 = \frac{\text{Net Expense}}{\text{Net Prem}} + \frac{\text{Net loss}}{\text{Net Prem}} = \frac{200 + \text{Net loss}}{1,000 - 220}$$

$$\text{Net Loss}_1 = 447.4$$

$$\text{Ceded Loss}_1 = 600 - 447.4 = 152.6$$

$$CR_2 = 0.86 = \frac{\text{Net Expense}}{\text{Net Prem}} + \frac{\text{Net loss}}{\text{Net Prem}} = \frac{200 + \text{Net loss}}{1,000 - 330}$$

$$\text{Net Loss}_2 = 376.2$$

$$\text{Ceded Loss}_2 = 600 - 376.2 = 223.8$$

$$ROE_1 = 67.4 / 500 = 0.1348$$

$$ROE_2 = 106.2 / 900 = 0.118$$

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT

Candidates were expected to calculate the marginal net cost of each of the two reinsurance programs by using the change in Net Underwriting Income from the Current, and using the difference between Ceded Losses and Ceded Premiums.

Candidates were then expected to ratio this cost to the marginal change in required capital, as disclosed by the 1-in-100 TV@R value, to arrive at a marginal ROE for each reinsurance program option.

Common mistakes included:

- Incorrect calculations of Net Loss or Net Underwriting Income (e.g. applying the Net Combined Ratio to gross premium instead of net premium, or assuming the expense ratio was fixed at 20% for each option).
- Calculations of Reinsurance program costs (e.g. using the Expected Ceded Premium as the "cost" of the reinsurance program, or calculating marginal cost as $(\text{Net Combined Ratio} - 80\%) \times \text{Gross Premium}$).
- Calculating the ROE of each option as Net UW Income divided by Required Capital and then calculating the marginal ROE as the difference in these ROEs.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 21	
TOTAL POINT VALUE: 1.75	LEARNING OBJECTIVE(S): C3, C4
SAMPLE ANSWERS	
Part a: 0.75 point	
<p><u>Sample 1</u> 1-in-100 TV@R for personal property = average of losses to simulation #709 down to #663 since $1,000 \times 0.01 = 10$ = 3,199,600</p> <p>Capital allocated: Personal auto = $10M \times 4.5M / (4.5M + 3.1996M) = 5.8445$ million Personal property = $10M \times 3.1996M / (4.5M + 3.1996M) = 4.1555$ million</p> <p><u>Sample 2</u> Prop 1-100 TVAR = $(6,933,000 + \dots + 836,000) / 10 = 3,199,600$</p> <p>Auto: $4.5 / (4.5 + 3.2) \times 10 = 5.84M$ Prop: $3.2 / (4.5 + 3.2) \times 10 = 4.16M$</p>	
Part b: 1 point	
<p><u>Sample 1</u> WTVAR Property = $(6,933 \times 0.5 + 5,606 \times 0.4 + 4,867 \times 0.1) / 10 = \\$6.1956M$ (near \$6.2M).</p> <p>Auto capital = $\\$10M \times 5 / (6.2 + 5) = \\$4.46M$ Property capital = $\\$10M \times 6.2 / 11.2 = \\$5.54M$</p> <p><u>Sample 2</u> WTVaR personal prop = $(0.001 \times 4867 + 0.004 \times 5,606 + 0.005 \times 6,933) / 0.01 = 6,195,600$</p> <p>Personal property alloc = $6,195,600 / (6,195,600 + 5,000,000) \times 10M = 5,533,960$ Auto = $10M - 5,533,960 = 4,466,040$ Auto gets the rest of the capital after allocating it to property.</p>	
EXAMINER'S REPORT	
<p>Candidates were expected to understand how to allocate a company total capital by line of business using the proportional allocation method with different risk measures (1-in-100 TV@R and 1-in-100 WTV@R here). For this, they were also expected to understand these risk measures and know how to calculate them.</p>	
Part a	
<p>Candidates were expected to know how to calculate 1-in-100 TV@R value for a company's line of business using a certain number of large loss simulations. They were also expected to be able to allocate the company capital proportionally by line of business using each line's TV@R.</p>	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Common mistakes included:

- Using incorrect number of simulations for the property TV@R calculation.
- Using the V@R instead of the TV@R for property (or a loss value near the V@R).
- Using the WTV@R instead of the TV@R for property.

Part b

Candidates were expected to know how to calculate 1-in-100 WTV@R value for a company's line of business using a predefined transformed probability distribution for one of the lines.

Candidates were expected to understand the WTV@R measure and know which loss simulations to use in the calculation with their right probability (capped probability for the last simulation).

Candidates were also expected to be able to allocate the company capital proportionally by line of business using each line's WTV@R.

Common mistakes included:

- Using inadequate weights for the WTV@R calculation.
- Forgetting to cap the probability of the last simulation.
- Using the TV@R instead of the WTV@R for auto.
- Using the V@R instead of the WTV@R for property (or a loss value near of the V@R).
- Using the 10 worst or the 13 given loss simulations in the property WTV@R calculation.
- Weighting the loss simulations with $F^*(x)$ instead of $f^*(x)$.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 22						
TOTAL POINT VALUE: 3			LEARNING OBJECTIVE(S): C1, C2			
SAMPLE ANSWERS						
Part a: 0.75 point						
<u>Sample 1</u>						
Strategy 1: $80 \times (0.08 - 0.05) = 2.4$ million in 1 st year						
Strategy 2: $200 \times 0.1 \times 0.15 = 3$ million in 1 st year						
On a 1-year basis, strategy 1 is preferred.						
<u>Sample 2</u>						
St1.						
Coupon Payout $80 \times 0.08 = 6.4$						
Bond yield = $80 \times 0.05 = 4$						
Cost = $6.4 - 4 = 2.4M$						
St2.						
Cost = $200M \times 10\% \times 15\% = 3M$						
Strategy 1 is preferred.						
<u>Sample 3</u>						
Cost of surplus note in 1 st year:						
$80 \text{ million} \times (8\% - 5\%) / 1.05 = 2.286 \text{ million}$						
Cost of reinsurance in 1 st year:						
$200 \text{ million} \times 10\% \times 15\% / 1.05 = 2.857 \text{ million} > 2.286 \text{ million}$						
Surplus note is preferred on one-year basis.						
Part b: 1.25 points						
<u>Sample 1</u>						
Option 1						
$\frac{2.4}{1.05} + \frac{2.4}{1.05^2} + \frac{2.4}{1.05^3} + \frac{2.4}{1.05^4} + \frac{2.4}{1.05^5} = 10.39$						
Option 2						
$\frac{3M}{1.05} + \frac{2.4}{1.05^2} + \frac{0.9}{1.05^3} = 5.81$						
Option 2 is preferred on a 5-year basis.						
<u>Sample 2</u>						
	Yr1	2	3	4	5	Total
#1 Cost	2.4	2.4	2.4	2.4	2.4	
#1 Discount	$2.4/1.05=2.29$	$2.4/1.05^2=2.18$	2.07	1.97	1.88	10.39

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

	Yr1	2	3	4	5	Total
#2 Cost	3M	$200 \times 15\% \times 8\% = 2.4$	0.9	0	0	
#2 Discount	$3/1.05 = 2.86$	$2.4/1.05^2 = 2.18$	0.78	0	0	5.81

5.81 < 10.39, so #2 strategy is preferred.

Sample 3

Cost of surplus note over 5 years:

$$80 \times (8\% - 5\%) / 1.05 + \dots + 80 \times (8\% - 5\%) / 1.05^5 = 10.39 \text{ million}$$

Cost of reinsurance over 5 years:

$$200 \times 10\% \times 15\% / 1.05 + 200 \times 8\% \times 15\% / 1.05^2 + 200 \times 3\% \times 15\% / 1.05^3 = 5.811 \text{ million}$$

The reinsurance is preferred on a 5-year basis.

Sample 4

Strategy 1

$$\text{Cost} = 80\text{M} - (0.08)(80\text{M})/1.05 + (0.08)(80\text{M})/1.05^2 + \dots + 0.08(80\text{M})/1.05^5 - (80)/1.05^5 = -10.4\text{M}$$

Strategy 2

$$0.15(0.10)(200) + 0.15(0.08)(200)/1.05 + 0.15(0.08)(200) / 1.05^2 = 6.1\text{M}$$

Strategy 2 preferred.

Part c: 1 point

Sample 1

- Reinsurance provides stability
 - Reinsurance frees up capital
 - Reinsurance adds to firm equity value
- Second paradigm: Frees up capital was used

Sample 2

- Is stable
- Surplus release
- Market value added

Sample 3

Three paradigms are stability, frees up capital, and adding values.
Frees up capital by using less cost to cover the possible exposures

Sample 4

- Stability – of earnings, capital, etc.
 - Decrease in required capital
 - Adds value to the firm
- This strategy will result in a decrease in required capital

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Sample 5

- Stabilize results
 - As a replacement for required surplus
 - Adds value to the firm
- As a replacement for required surplus

Sample 6

- Reinsurance adds stability to results
 - Reinsurance creates firm value
 - Reinsurance helps manage surplus
- Paradigm 3 was used to evaluate the structure of Strategy 2

EXAMINER'S REPORT

Candidates were expected to demonstrate general knowledge of how to assess the costs associated with strategies used to mitigate insurance risk. They were expected to demonstrate awareness of common Enterprise Risk Management concepts underlying capital management.

Part a

Candidates were expected to know how to calculate the 1st year cost of issuing a surplus note and the cost of purchasing reinsurance, and to be able to identify which strategy would be preferred based on the cost of each.

Common mistakes included:

- Calculating only the value of the coupon payment as the associated surplus note cost for strategy 1.
- Using the economic margin on the reinsurance contract incorrectly to calculate the cost of strategy 2.

Part b

Candidates were expected to know how to calculate the cost of each of the two strategies on a 5-year basis, and be able to identify that the lower cost option would be preferred. Candidates were expected to be able to discount the cost of each strategy using the bond yield that was provided.

Common mistakes included:

- Not discounting the cost of each strategy.
- Using the coupon rate as the discount rate.
- Using the surplus note principal repayment as part of the cost of employing strategy 1.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Part c

Candidates were expected to be able to identify the three paradigms commonly used to assess the economic value of a reinsurance contract and to be able to identify which of the three was used to evaluate the quota share reinsurance strategy

Common mistakes included:

- Providing two items that were both related to the same paradigm.
- Incorrectly identifying which paradigm was used to evaluate the quota share reinsurance strategy.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 23	
TOTAL POINT VALUE: 2	LEARNING OBJECTIVE(S): C1, C5, C6
SAMPLE ANSWERS	
<i>Sample responses are separated into the four main characteristics listed in the question:</i>	
Characteristic i	
<ul style="list-style-type: none">• Work and scheduling conflicts with employees' regular jobs. Have some dedicated employees to work solely on ERM.• This introduces risk of delay of ERM implementation as employees may say they are too busy to do their ERM work on time. Have dedicated full time employees do ERM work.	
Characteristic ii	
<ul style="list-style-type: none">• Results from cat models can vary greatly from version to version. Using multiple models is superior.• Only one model used. Should use multiple models to reduce internal systemic risk.	
Characteristic iii	
<ul style="list-style-type: none">• Model should be parameterized by people with deep knowledge of subject. Correlation requires deep knowledge of multiples lines, not just one. Have ERM team consider correlations themselves in addition when selecting.• Parameters are built using different types of assumptions. Roundtable with actuary and LOB leaders to arrive at a consensus on parameters and parameter variance.• Each leader does not have enough of the overall firm knowledge to sufficiently estimate parameters. Should be owned at the corporate level.• Because of its high political sensitivity, individual business leaders may skew the parameters in their favor. Correlation assumptions should be owned by the CRO or ERM department.	
Characteristic iv	
<ul style="list-style-type: none">• Significant impairment to operations could happen well before default. Consider metrics such as inability to service renewals, or ratings downgrades as thresholds.• Default risk too far out in the tail, tail is most uncertain and unreliable. Use different threshold. Capital necessary to maintain rating level.• Weakness: Capital set only to prevent default mostly protects policyholders – shareholders may have different risk tolerance. Improvement: Look at range of requirements necessary to not only recover but thrive from a CAT.	
EXAMINER'S REPORT	
Candidates were expected to know concepts of ERM and be able to identify weaknesses and know possible improvements. Candidates were expected to either explain why their weakness was a weakness or explain why their recommendation was an improvement to the weakness.	
Characteristic i	
<ul style="list-style-type: none">• Candidates were expected to recognize that part time employees cannot dedicate time to the ERM process and that full time/dedicated employees were necessary.	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

Characteristic ii

- Candidates were expected to know that there is uncertainty in the catastrophe model such as data quality and model/parameter risk and that using multiple models would address this issue. Candidates were expected to state why having one model was a weakness or what improvement multiple models would bring.

Characteristic iii

- Candidates were expected to know that individual Line of Business leaders are inexperienced or biased in setting correlations, and that assumptions be held by the CEO/CUO/CRO or in collaboration among them.

Characteristic iv

- Candidates were expected to provide a weakness that emphasizes the uncertain nature of modeling in the tail, protecting policyholders but not shareholders, or that losses lower than default can have negative impacts on the firm.

Common mistakes included:

- Not providing a weakness, or not explaining why the weakness identified was a weakness.
- Not explaining why the proposed solution was an improvement.

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

SPRING 2018 EXAM 7, QUESTION 24	
TOTAL POINT VALUE: 1	LEARNING OBJECTIVE(S): C8
SAMPLE ANSWERS	
Part a: 0.5 point	
<u>Sample Response</u>	
<ul style="list-style-type: none"> • Keep writing business in order to maintain same volume even though prices have decreased. The implication is you are writing more business at inadequate rates since market has softened. • The firm decreases the price and/or expands coverage to maintain the market during the soft market. This action would result in a price adequacy drop. When the cycle hits the bottom, the firm will recognize the increased loss from the increased exposure. This may result in a downgrade. • The strategy is to decrease price as to not lose volume. This may result in underpricing. • Naïve underwriting cycle strategy. This would result in taking on additional exposures at a lower price which could hurt results. • They will sell business at unprofitable rates. This may lead to default once claims start to come in. • Company drops prices to prevent losing customers during a soft market. When market over-corrects, company may not have the capacity to "cash in" on the higher pricing and greater profits. • The company would have a "plan" premium target and will strive to make that target no matter what. However, in a soft market, prices are low and the company's rates will be insufficient to be profitable which will cause significant issues down the road. 	
Part b: 0.5 point	
<u>Sample Response</u>	
<ul style="list-style-type: none"> • Change underwriting incentives to allow underwriters to write fewer premiums while not being worried about job safety or bonuses. • Align underwriter incentive with meeting objective of cycle management strategy rather than growth in plan. • Maintain underwriter discipline – do not write unprofitable business even if it means giving up market share. • Maintain intellectual property – retain top talent and continue investment in data systems. • Invest in intellectual property – retain and train people to be ready to write more when cycle turns. • Maintain the company's top talent and grow their skills. Talent such as underwriters or actuary. • Maintain presence in core market. When the soft market turns, underwriter can start writing business again at better pricing. • You can educate the owner to let them know premium volumes may drop. • Owner education – educate the owner to understand that decreased premium volume and increased overhead expense is expected in soft markets when practicing effective cycle management. 	

EXAM 7 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

- Educate owners to understand the strategy and impact, not to increase market share in soft market.
- Scenario planning – prepare for possible scenarios of changes to the market so you have a pre-determined plan to act upon.
- Market overreaction – the company reduces its writings in soft markets and then when the market hardens it can aggressively write more business at higher rates since it will have sufficient capital from not writing in soft years.
- Investment strategy – Invest in taxable bonds during soft market with low/ no profits. Invest in non-taxable bonds in hard market.
- Consider agent theory so that management goals are aligned with owner's goals.
- Also plan ahead on market overreaction. Keep surplus healthy in anticipation of capitalizing when prices improve. Windfalls reaped will compensate for losses in soft market.

EXAMINER'S REPORT

Candidates were expected to understand the definition of a soft market and the strategies a company could follow during a soft market, along with their implications.

Part a

Candidates were expected to explain the strategy and provide a negative implication of this strategy.

Common mistakes included:

- Not providing adequate explanation of the strategy or repeating the wording in the question.
- Providing only a negative implication, without an explanation of the strategy.

Part b

Candidates were expected to know features of alternative underwriting cycle management strategies.

Common mistakes included:

- Providing only one strategy.
- Not providing adequate explanation of the strategies listed.