Relative Unpaid Claims Loss Reserving

Bertram A. Horowitz, FCAS, MAAA

"Let no one say that I have said nothing new; the arrangement of the subject is new."

—Blaise Pascal

Abstract: This paper derives an elementary Relative Unpaid Claims Loss Reserving Model using only: (i) accident year incremental losses that were paid during the same calendar year as the accounting date; (ii) relativities of successive accident year unpaid losses as of the accounting date; and (iii) unpaid losses for the oldest included accident year as of the accounting date. Methods to apply the Model are presented along with considerations and techniques to improve accuracy. Several methods derived from the Model are applied to the CAS loss reserve data base historical experience and the resulting unpaid claim estimates are compared to indications using traditional loss reserving methods. Performance accuracy of competing methods is evaluated using a retrospective hindsight test of subsequent emergence. Advantages of the Relative Unpaid Claims Loss Reserving Model include that it requires less data and fewer assumptions than traditional chain ladder methods and results in unpaid claim estimates that empirically appear at least as accurate as estimates derived from comparable generally accepted actuarial loss reserving methods.

Keywords: loss reserve; reserving; unpaid claims; IBNR; recursive model; relative

1. INTRODUCTION

1.1 Historical Background

As expounded upon by Friedland [4], basic loss reserving methods are fundamentally rooted in loss development triangles and associated loss development factors. After appropriate investigation, traditional loss reserve analyses typically proceed with compilations of historically based accident year[1] loss development triangles intended to be representative of expected future development. Loss development factors derived from these historical development patterns are applied to accident year experience as of the valuation date to extrapolate historical development into the future and, thereby, estimate ultimate accident year losses. Unpaid loss estimates as of a particular accounting date are indirectly calculated by subtracting cumulative loss payments through the accounting date from estimated accident year ultimate losses.

Even where "expected loss" is introduced to improve the accuracy of estimated ultimate losses, commonly applied methods (e.g., Bornhuetter-Ferguson, Cape Cod) also require loss development factor selections. Basic frequency-severity (counts & averages) methods are similarly organized into development triangles and require selections for some combination of loss

[1] Accident year is referenced throughout this paper since it is the most common categorization of historical data. Techniques described in this paper are also applicable to data categorized by other time intervals including policy year, underwriting year, report year and fiscal year. Similarly, the techniques are applicable to monthly, quarterly and biannual data.

development factors, frequency trend, severity trend, and disposal rates.

1.2 Objective

This paper presents a straightforward and robust Relative Unpaid Claims Loss Reserving model conceived from a different perspective than traditional chain ladder loss development models. Methods to apply the relative unpaid claims model use estimated ratios of unpaid claims[2] as of the accounting date for successive accident years and an estimate of unpaid claims for the oldest accident year to directly estimate unpaid losses for each accident year. These methods are relatively easy to apply and, optimally, improve the accuracy of unpaid claim estimates while requiring less data and fewer assumptions than traditional chain ladder loss development triangle methods.

1.3 Outline

The remainder of this paper presents a framework and describes techniques to estimate unpaid claims from relationships derived in Section 2:

- Section 2 presents Relative Unpaid Claims Loss Reserving basics;
- Section 3 provides an illustrative example;
- Section 4 discusses measures of relative unpaid losses at common maturities;
- Section 5 addresses unpaid losses for the oldest included accident year;
- Section 6 explores empirical evidence using the CAS loss reserve data base to compare results of methods that apply the Relative Unpaid Claims Loss Reserving model to the results of several generally accepted actuarial loss reserving methodologies;
- Section 7 summarizes relevant results; and
- Section 8 presents the main conclusions and areas for future research.

2. RELATIVE UNPAID CLAIMS LOSS RESERVING BASICS

We derive a relative unpaid claims model from definitions.

2.1 Definitions

For consecutive accident years *m* through *n* ($n>m$), define:

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[2] The techniques presented are applicable to loss dollars, claim counts, ALAE (DCCE), and loss & ALAE (DCCE) combined.

$U_{i,j}$ = accident year *i* unpaid losses as of year-end *j*, where $j \geq i$,

$p_{i,j}$ = accident year *i* payments during calendar year *j*, where $j \geq i$.

As of accounting date year-end *d*, define the ratio of unpaid loss at common maturities:

$r_i = \dfrac{U_{i,d}}{U_{i-1,d-1}}$, where $m + 1 \leq i \leq n$, $d \geq n$

r_i equals the **relativity** of accident year i unpaid losses as of accounting date year-end d to accident year i-1 unpaid losses as of accounting date year-end d-1.

2.2 Relative Unpaid Claims Loss Reserving Model ("Model")

Beginning with initial value $U_{m,d}$, each $U_{i,d}$ ($m + 1 \leq i \leq n$, $d \geq n$) may be computed using the recursive algorithm:

$$U_{m+1,d} = r_{m+1}[U_{m,d} + p_{m,d}]$$
$$U_{m+2,d} = r_{m+2}[U_{m+1,d} + p_{m+1,d}]$$
$$\dots$$
$$U_{n,d} = r_n[U_{n-1,d} + p_{n-1,d}]$$

Proof:

The proof follows directly from definitions.

It is self-evident that

$$U_{i-1,d-1} = U_{i-1,d} + p_{i-1,d} \tag{2.1}$$

From the definition of r_i

$$U_{i,d} = r_i U_{i-1,d-1} \tag{2.2}$$

Substituting the expression for $U_{i-1,d-1}$ of (2.1) into (2.2) gives us

$$U_{i,d} = r_i[U_{i-1,d} + p_{i-1,d}], \text{ where } m + 1 \leq i \leq n,\ d \geq n \tag{2.3}$$

Given a base value for $U_{m,d}$, recursive application of (2.3) commencing with i=m+1 and ending with i=n results in the Model algorithm. Q.E.D.

A closed-form expression for each $U_{i,d}$ is presented in Appendix A.

The Model demonstrates that, in order to determine accident year *m* through *n* unpaid losses as of accounting date year-end *d*, it is sufficient to know: (i) $p_{i,d}$, the incremental paid losses during calendar year *d* for each accident year *i*=*m* through *i*=*n*-1; (ii) r_i, the ratio of accident year *i* unpaid losses as of accounting date year-end *d* to accident year *i*-1 unpaid losses as of accounting date year-end *d*-1 for each *i*=*m*+1 through *i*=*n*; and (iii) $U_{m,d}$, the unpaid losses of accident year *m* as of accounting date year-end *d*.

The Model specifies an unpaid claims algorithm that provides an exact representation of unpaid losses (i.e., perfectly accurate Model parameters result in perfectly accurate unpaid losses for each accident year; whereas, in a traditional chain ladder model setting, the most accurate loss development factor selections are not expected to result in perfectly accurate unpaid loss estimates for each accident year). Generally, model risk is the risk that the methods are not appropriate to the circumstances or the models are not representative of the specified phenomenon. Since the Model provides an exact representation of unpaid losses, the second aspect of model risk is eliminated. For application of the Model, we refer to items (i) - (iii) above: (i) $p_{i,d}$ will typically be known as part of the historical data base for the vast majority of loss reserve analyses; (ii) r_i will typically be unknown and estimated; and (iii) $U_{m,d}$ will typically be unknown and estimated using methods analogous to tail factor development methods. Various methods to derive unpaid claims estimates using the Model will be explored in greater depth. However, we immediately proceed to a simple Relative Unpaid Claims Loss Reserving illustrative example.

3. ILLUSTRATIVE EXAMPLE METHOD

This section presents an example to illustrate use of the Model's algorithm to estimate unpaid losses from a large business segment of actual Other Liability – Occurrence experience[3]. Though the term 'loss' is used for convenience, examples presented in this paper are actually comprised of combined loss & ALAE (DCCE) experience. All loss dollar data presented throughout this paper are displayed in thousands of dollars (i.e., $000 omitted).

3.1 Rudimentary Assumptions

For this example: (i) $p_{i,d}$, the incremental paid losses during calendar year *d*=1997 for each accident year *i*=*m*=1988 through *i*=*n*-1=1996, are known; (ii) r_i, the ratio of accident year *i* unpaid losses as of accounting date year-end *d*=1997 to accident year *i*-1 unpaid losses as of accounting date year-end *d*-1=1996 for each *i*=*m*+1=1989 through *i*=*n*=1997 are assumed to

[3] CAS Loss Reserve Data Base [7]: Other Liability Data Set; NAIC Company Code 1767

equal the ratio of corresponding case reserves; and (iii) $U_{m,d}$, unpaid losses of accident year *m*=1988 as of accounting date year-end *d*=1997 is assumed to equal the corresponding company filed loss reserves (including IBNR).

By utilizing the ratio of case reserves as of the latest common maturities, (ii) assumes that this ratio is an accurate proxy for the relativity of all (including IBNR) unpaid losses as of the most recent common maturities. By accepting the company filed loss reserves (including IBNR) as of year-end *d*=1997 for the oldest included accident year *m*=1988, (iii) assumes that these filed loss reserves accurately provide for the corresponding unpaid claims.

Table 3.1 displays these assumptions. Table 3.1, Column (4) derives each estimated r_i as specified in (ii) above. Table 3.1, Column (5) displays the company filed loss reserves (including IBNR) as of 12/31/97 for oldest accident year 1988 as in (iii) above.

TABLE 3.1

ASSUMPTIONS SELECTION

(1)	(2)	(3)	(4)= (3)/[Prior(2)]	(5)
Accident Year	Case Reserves as of 12/31/96	Case Reserves as of 12/31/97	Selected Ratio Unpaid Loss	Selected Unpaid Loss of Oldest Accident Year as of 12/31/97
i			Selected r_i	Selected $U_{1988,1997}$
1988	1,588	116		1,048
1989	2,838	1,419	0.8935768	
1990	4,883	1,436	0.5059901	
1991	7,016	3,282	0.6721278	
1992	23,466	11,991	1.7090935	
1993	31,248	15,482	0.6597631	
1994	56,994	46,505	1.4882552	
1995	66,826	55,399	0.9720146	
1996	54,941	70,761	1.0588843	
1997		61,839	1.1255529	

(1) m=1988; n=1997
(2), (3) CAS Loss Reserve Data Base [7]
(5) CAS Loss Reserve Data Base [7] = company filed loss reserves (including IBNR) as of 12/31/97

3.2 Derive Unpaid Claims Estimate

Table 3.2, Column (4) uses Table 3.1 assumptions to apply the Model and derive estimated unpaid losses as of accounting date 12/31/97[4] for each accident year 1988 through 1997. The Table 3.2, Column (4) accounting date 12/31/97 indicated total accident year 1988 through 1997 unpaid losses equals $853,442.

TABLE 3.2

INDICATED UNPAID LOSSES

(1)	(2)	(3)	(4)	(5)
Accident Year	Case Reserve Ratio Unpaid	Incremental Paid Loss During 1997	Indicated Unpaid Loss as of 12/31/97	Actual Emergence
i	Selected r_i	$p_{i,1997}$	Indicated $U_{i,1997}$	
1988		2,064	1,048	1,048
1989	0.8935768	5,085	2,781	2,229
1990	0.5059901	3,432	3,980	4,875
1991	0.6721278	13,032	4,982	8,939
1992	1.7090935	17,241	30,787	27,175
1993	0.6597631	23,924	31,687	38,236
1994	1.4882552	56,447	82,764	75,947
1995	0.9720146	77,480	135,315	130,558
1996	1.0588843	72,104	225,325	216,789
1997	1.1255529	21,098	334,772	309,458
Total			**853,442**	815,254

(1) m=1988; n=1997
(2) Table 3.1, Column (4)
(3) CAS Loss Reserve Data Base [7]
(4) d=1997
For $i = m = 1988$: Table 3.1, Column (5)
For $1989 \le i \le 1997$: (2)x[Prior (3)+Prior(4)]
(5) Computed from CAS Loss Reserve Data Base [7]
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year + company filed loss reserves (including IBNR) nine years subsequent to accident year

[4] All examples in this paper as of accounting date 12/31/97 estimate unpaid losses as of valuation date 12/31/97.

3.3 Retrospective Testing

For the purposes of examples throughout this paper, the term "actual emergence" is defined as cumulative losses paid subsequent to 12/31/97 through nine[5] years subsequent to the accident year added to company filed loss reserves (including IBNR) as of nine years subsequent to the accident year. The Table 3.2, Column (5) total actual emergence equals $815,254. Comparison of the Table 3.2, Column (4) indicated unpaid losses with the Table 3.2, Column (5) actual emergence provides a retrospective test of indicated unpaid claim estimate accuracy. This retrospective test demonstrates that the method results in accounting date 12/31/97 indicated total unpaid losses within 5% of the total actual emergence.

3.4 Initial Observations

Implementation of the Model using the method described in this illustrative example has several advantages over traditional chain ladder loss development reserving methods:

- The method is more efficient to apply;
- The method only requires experience from the most recent calendar year. As such, this method requires less data and information than chain ladder loss development methods since there is no need to produce loss development triangles and no need to select loss development factors;
- It is not necessary to understand or analyze how possible changes in claim payment patterns, case reserve adequacy or other potential distortions have wended their way through an entire historical loss development triangle. As such, it is unnecessary to attempt to adjust for these changes over an entire historical loss development triangle; and
- Given the most recent calendar year payments by accident year, all that is required to effectively employ this method is: case reserves for accident year i at accounting date year-end d divided by case reserves for accident year i-1 at accounting date year-end d-1 reasonably estimate the corresponding ratio of total unpaid losses; and a reasonable estimate of unpaid losses for the oldest included accident year m as of accounting date year-end d.

[5] Nine years subsequent to the accident year is the maximum number of development years available from the CAS loss reserve data base.

4. ESTIMATING RELATIVITY OF UNPAID LOSSES r_i

Selection of appropriate r_i is critical for successful application the Model. It can be useful to conceptualize appropriate r_i as reasonable measures of relative exposure to unpaid losses. For example, the case reserve ratio assumption (ii) in Section 3 is tantamount to the assumption that case reserve ratios measure the corresponding relative exposure to total (including IBNR) unpaid losses.

While the Section 3 illustrative example uses the ratio of case reserves to estimate r_i, several issues may cause case reserve ratios, or other measures, to be a distorted measure of relative unpaid losses. Distortions may result from three general areas: internal (e.g., shifts in mix of business, changes in claim settlement procedures, changes in case reserve adequacy); external (e.g., law changes, inflation, social influences); and credibility (i.e., randomness or sparseness of data renders it unrepresentative of the future). Potential distortions may occur in isolation or concurrently. In Section 2 of their paper "Accident Year/Development Year Interactions" [2], Clark and Rangelova discuss internal and external considerations in the context of loss development patterns. Generally, these considerations are also pertinent to estimating r_i. Credibility distortions arise when potential r_i measures do not have sufficient predictive power to reasonably measure the relativity of unpaid losses.

The following subsections discuss r_i characteristics and potential r_i measures or proxies.

4.1 Reproduction of Actual Emerged Losses

Pursuant to the Model, incremental calendar year d payments for each accident year together with foreknowledge of the actual ratios of unpaid losses r_i and foreknowledge of unpaid losses for the oldest included accident year $U_{m,d}$ determine unpaid losses for all accident years as of year-end d. It is instructive to derive unpaid losses for the Section 3 example based upon the r_i and $U_{m,d}$ implicit in actual emergence. Table 4.1.1 uses foreknowledge of the actual emergence to solve for r_i and $U_{m,d}$.

TABLE 4.1.1

ASSUMPTIONS SELECTION

(1) Accident Year	(2) Unpaid Loss as of 12/31/97 Actual Emergence	(3) Incremental Paid Loss During 1997	(4)= (2)+(3) Unpaid Loss as of 12/31/96 Emergence	(5)= (2)/[Prior (4)] Selected Ratio Unpaid Loss	(6) Selected Unpaid Loss of Oldest Accident Year as of 12/31/97
i				Selected r_i	Selected $U_{1988,1997}$
1988	1,048	2,064	3,112		1,048
1989	2,229	5,085	7,314	0.7162596	
1990	4,875	3,432	8,307	0.6665299	
1991	8,939	13,032	21,971	1.0760804	
1992	27,175	17,241	44,416	1.2368577	
1993	38,236	23,924	62,160	0.8608610	
1994	75,947	56,447	132,394	1.2217986	
1995	130,558	77,480	208,038	0.9861323	
1996	216,789	72,104	288,893	1.0420644	
1997	309,458	21,098	330,556	1.0711855	

(1) m=1988; n=1997
(2) Table 3.2, Column (5)
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year + company filed loss reserves (including IBNR) nine years subsequent to accident year
(3) Table 3.2, Column (3)
(4) cumulative losses paid subsequent to 12/31/96 through nine years subsequent to accident year + company filed unpaid losses (including IBNR) nine years subsequent to accident year
(6) CAS Loss Reserve Data Base [7] = company filed loss reserves (including IBNR) as of 12/31/97

Table 4.1.2 inputs the resulting r_i and $U_{m,d}$ into the Model and, as we would expect, demonstrates that actual emergence is indeed reproduced for each accident year.

TABLE 4.1.2

INDICATED UNPAID LOSSES

(1)	(2)	(3)	(4)	(5)
Accident Year	Selected Ratio Unpaid	Incremental Paid Loss During 1997	Indicated Unpaid Loss as of 12/31/97	Actual Emergence
i	Selected r_i	$p_{i,1997}$	Indicated $U_{i,1997}$	
1988		2,064	1,048	1,048
1989	0.7162596	5,085	2,229	2,229
1990	0.6665299	3,432	4,875	4,875
1991	1.0760804	13,032	8,939	8,939
1992	1.2368577	17,241	27,175	27,175
1993	0.8608610	23,924	38,236	38,236
1994	1.2217986	56,447	75,947	75,947
1995	0.9861323	77,480	130,558	130,558
1996	1.0420644	72,104	216,789	216,789
1997	1.0711855	21,098	309,458	309,458
Total			815,254	815,254

(1) m=1988; n=1997
(2) Table 4.1.1, Column (5)
(3) Table 4.1.1, Column (3)
(4) d=1997
For i = m = 1988: Table 4.1.1, Column (6)
For 1989 ≤ i ≤ 1997: (2)x[Prior (3)+Prior(4)]
(5) CAS Loss Reserve Data Base [7]
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year + company filed loss reserves (including IBNR) nine years subsequent to accident year

4.2 Case Reserves

Section 3 uses the ratio of case reserves as a rudimentary measure of r_i under the assumption that case reserve ratios are a reasonable estimate of relative total (including IBNR) unpaid losses (or, alternatively, relative total unpaid loss exposure). The following provides several advantages and potential distortions in the use of case reserve ratios as proxies for r_i:

Advantages -

- Case reserves are typically readily available.
- Case reserves reflect actual loss experience.
- The ratio of case reserves at common maturities measures the implicit aggregate relative case reserves established by claims personnel acting on behalf of the insuring entity. Accordingly, if claims personnel have behaved consistently, the ratio of case reserves at common maturities as a measure of the ratio of all unpaid losses (including IBNR) is intuitively appealing.

Potential Distortions -

- Non-homogenous mix of business.
- Case reserves may have established at different levels of adequacy. This might occur due to changing conditions (e.g., claims personnel practices) or external conditions (e.g., inflation).
- Although case reserves may be evaluated at a common time maturity, such common time maturity may correspond to different stages of development and, thereby, distort case reserve ratios as an appropriate measure of relative total unpaid losses.
- The relativity of IBNR losses may be different than the corresponding case reserve ratio.
- Sparse case reserve experience may reduce the credibility of the case reserve ratio as a reasonable measure of relative unpaid exposure. This may be especially true for: relatively small books of business with relatively low volume; older accident years which are more fully developed and have relatively few remaining case reserves; and recent accident years for slow developing lines of business where only relatively few (or no) case reserves have yet been established.

It may be possible to partition, aggregate or adjust data to eliminate or mitigate distortions in the use of case reserve ratios as a measure of relative total unpaid losses. As discussed by Gross [5], actuaries may use claims level predictive analytics to build their own models of unbiased case reserves based upon detailed objective information about claims and exposure. In general, when considering r_i candidates (case reserve ratios or otherwise), it is prudent to weigh strengths and weaknesses of competing measures.

4.3 Calendar Year d Reported Emergence

While the ratio of case reserves at common maturities is an obvious candidate to estimate r_i, we may not have taken full advantage of all available information. To estimate each r_i, we have not yet made use of reported emergence during calendar year *d*.

Appendix B, Sheet 1 displays historical incremental paid losses and case reserves for the Section 3 business segment. Appendix B, Sheet 2 displays the case reserves to left of the corresponding one year reported[6] losses emerged along with the resultant underlined one-year loss development factor[7] displayed underneath. The one-year loss development factors are case reserve development factors that develop case reserves as of year-end to subsequent one year reported emergence (i.e., to payments during the next calendar year plus case reserves as of the next calendar year-end).

As a result of reversion to the mean, if the one-year loss development factors as of year 2 are samples from the same random variable, then an average of the sample loss development factors is generally a more accurate estimate of the future one-year loss development factor as of year 2 than simply repeating the most recent value. The same is true for one-year loss development factors as of years 3, 4, . . . The final underlined row of Appendix B, Sheet 2 displays the (up to) three most recent years dollar weighted average of one-year loss development factors.

The dollar weighted average one-year loss development factors from the final underlined row of Appendix B, Sheet 2 are selected to derive Table 4.3.1, Column (5) estimates of unpaid loss as of 12/31/97 reported as of 12/31/98 for the numerator of r_i. Table 4.3.1, Column (6) displays estimated r_i that incorporate 12/31/96 unpaid losses reported emergence during calendar year 1997.

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[6] One year reported losses for accident year *i* as of year *x* is defined as: accident year *i* incremental losses paid during maturity year *x* plus accident year *i* case reserves as of maturity year-end *x*.

[7] One-year loss development factor for accident year *i* as of *x* is defined as: one year reported losses for accident year *i* as of year *x* divided by accident year *i* case reserves as of maturity year-end *x*-1.

TABLE 4.3.1

ASSUMPTIONS SELECTION

(1) Accident Year	(2) Unpaid Loss as of 12/31/96 Reported as of 12/31/97	(3) Case Reserves as of 12/31/97	(4) Selected One Year Development Factor	(5)= (3)x(4) Estimated Unpaid Loss as of 12/31/97 Reported as of 12/31/98	(6)= (5)/[Prior (2)] Selected Ratio Unpaid Loss	(7) Selected Unpaid Loss of Oldest Accident Year as of 12/31/97
i					Selected r_i	Selected $U_{1988,1997}$
1988	2,180					1,048
1989	6,504	1,419	1.3727960	1,948	0.8935768	
1990	4,868	1,436	1.6909393	2,428	0.3733378	
1991	16,314	3,282	1.3999528	4,595	0.9438465	
1992	29,232	11,991	1.7282284	20,723	1.2702701	
1993	39,406	15,482	1.2571046	19,462	0.6657941	
1994	102,952	46,505	1.4460186	67,247	1.7065192	
1995	132,879	55,399	1.6082550	89,096	0.8654103	
1996	142,865	70,761	1.8627350	131,809	0.9919475	
1997		61,839	2.7249017	168,505	1.1794715	

(1) m=1988; n=1997
(2) Appendix B, Sheet 2; One Year Reported final diagonal
(3) Appendix B, Sheet 2; final diagonal
(4) Appendix B, Sheet 2; Wt'd Avg. Dev. Factor
(7) CAS Loss Reserve Data Base [7] = company filed loss reserves (including IBNR) as of 12/31/97

Table 4.3.2 inputs Table 4.3.1 assumptions into the Model to derive estimated unpaid losses as of 12/31/97. The Table 4.3.2, Column (4) total unpaid loss estimate of $799,986 is closer to the Column (5) actual emergence of $815,254 than the Table 3.2, Column (4) total unpaid loss estimate of $853,442. Indeed, the Table 4.3.2, Column (4) total unpaid claim estimate has narrowed the retrospective test accuracy from within 5% to within 2% of the actual emergence. Nonetheless, one should not generally presume that incorporating one year reported emergence during calendar year *d* will necessarily yield more accurate unpaid claim estimates than use of more rudimentary assumptions such as (ii) from Section 3.1.

TABLE 4.3.2

INDICATED UNPAID LOSSES

(1)	(2)	(3)	(4)	(5)
		Incremental	Indicated	
Accident	Selected Ratio	Paid Loss	Unpaid Loss	Actual
Year	Unpaid Loss	During 1997	as of 12/31/97	Emergence
i	Selected r_i	$p_{i,1997}$	Indicated $U_{i,1997}$	
1988		2,064	1,048	1,048
1989	0.8935768	5,085	2,781	2,229
1990	0.3733378	3,432	2,937	4,875
1991	0.9438465	13,032	6,011	8,939
1992	1.2702701	17,241	24,190	27,175
1993	0.6657941	23,924	27,584	38,236
1994	1.7065192	56,447	87,900	75,947
1995	0.8654103	77,480	124,919	130,558
1996	0.9919475	72,104	200,770	216,789
1997	1.1794715	21,098	321,847	309,458
Total			**799,986**	815,254

(1) m=1988; n=1997
(2) Table 4.3.1, Column (6)
(3) Table 3.2, Column (3)
(4) d=1997
For i = m = 1988: Table 4.3.1, Column (7)
For 1989 ≤ i ≤ 1997: (2)x[Prior (3)+Prior(4)]
(5) Computed from CAS Loss Reserve Data Base [7]
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year + company filed loss reserves (including IBNR) nine years subsequent to accident year

While obviously not the complete foreknowledge of Section 4.1, incorporating actual calendar year *d* one year reported emergence into estimated r_i includes additional loss experience available as of the valuation date that reflects more mature emergence toward the actual value of r_i than merely estimating r_i as case reserve ratios of Section 3.1. As such, the credibility of resulting r_i may be increased. It should also be noted that this procedure reintroduces a form of the loss development factor approach, albeit, for only one development year.

The foregoing procedure employs calendar year *d* reported emergence in the context of an incurred development method framework. More generally, this approach is applicable in the context of any loss reserving methodology that implicitly estimates accident year age-to-age development by calendar year.

4.4 Steady State Value for r_i = 1 + trend rate

A steady state system is defined herein as the same real (i.e., without consideration of frequency or severity trend) unpaid claim exposure as of common maturities for each accident year. In a steady state system, $r_i = 1 +$ (net impact of frequency and severity trend between $U_{i,d}$ and $U_{i-1,d-1}$). Consequently, if there were no unpaid frequency trend and no unpaid severity trend, steady state r_i would equal 1 for each *i*. These are important benchmark properties of r_i to bear in mind while considering appropriate r_i. To the extent that an indicated r_i moves further away from 1 (or, more precisely, 1 + unpaid expected trend rate), it is worthwhile to confirm that such r_i are reasonable and that the accident year-over-year indicated change in unpaid loss exposure is warranted. r_i would be expected to deviate from steady state values if there were a significant change in the expected unpaid loss volume between successive accident years at common maturities. Note that the actual r_i derived from the actual emergence of the Section 3 example fall within a range from .66 to 1.24 as displayed in Table 4.1.1, Column (5).

Similar steady state properties are absent from chain ladder development methods since there is no universal steady state CDF value. Steady state r_i properties remain valid regardless of development period length. On the other hand, the greater the expected development from a particular maturity, the higher the corresponding indicated CDF will be as of that maturity. CDFs from early maturities for slow developing lines of business are typically significantly greater than 1. Under near steady state conditions, indicated CDFs for long tailed lines may also be highly leveraged. While the r_i implicit in actual emergence from the Section 3 other liability example cluster near unity, the corresponding actual emergence accident year 1997 one year-to-ultimate incurred development CDF[8] equals 3.986 and the corresponding actual emergence one year-to-ultimate payment development CDF[9] equals 15.668.

4.5 Earned Premium

As a result of the relatively high volume in the numerator and denominator, the ratio of successive accident year earned premium may provide stability and credibility to corresponding r_i indications. Initially, it is preferable to set all earned premium to a common rate adequacy level before estimating r_i using earned premium as this would normally be expected to provide a more accurate measure of relative exposure than unadjusted earned premium[10]. In addition to

[8] Computed as (21,098 + 309,458)/(21,098 + 61,839) = 3.986 derived from Table 3.1 and Table 4.1.2

[9] Computed as (21,098 + 309,458)/21,098 = 15.668 derived from Table 4.1.2

[10] Pure Premium, the provision in the premium for loss & DCCE, would typically be an even more accurate basis for estimating r_i.

inconsistent premium adequacy, potential weaknesses of r_i based upon earned premium ratios are: they measure relative total accident year exposure rather than relative unpaid loss exposure; actual loss experience is not directly reflected; and expected unpaid losses are not directly considered. While the relative high volume of earned premium may add stability and credibility, a countervailing consideration is that resulting indicated r_i might suffer from reduced credibility as a result of potential earned premium ratio weaknesses. Table 4.5 uses earned premium from the Section 3 business segment to derive indicated r_i. For the final selection of r_i, it would typically be appropriate to complement earned premium ratio indications with other r_i measures since estimated r_i based solely upon earned premium would ignore the impact of recent loss experience through the valuation date.

TABLE 4.5

(1)	(2)	(3)= (2)/[Prior (2)]
Accident Year	Earned Premium	Indicated Ratio Unpaid Loss
i		Indicated r_i
1988	138,743	
1989	163,183	1.1761530
1990	162,184	0.9938780
1991	177,393	1.0937762
1992	197,770	1.1148692
1993	225,434	1.1398797
1994	267,578	1.1869461
1995	318,426	1.1900306
1996	363,402	1.1412447
1997	400,300	1.1015349

(2) CAS Loss Reserve Data Base [7]

4.6 Unpaid Claim Counts and Severity Indices

Where claim counts are available, their use may result in more accurate r_i estimates than other basic measures. When considering the use of claim counts, it is important that the definition and treatment of claim counts has been consistent. Potential claim count inconsistencies include, but are not limited to, changes in claim processing systems; treatment of incident claims; proportion of claims closed without payment; method of recording number of claims versus number of claimants; and time to establish claims. It may be possible to adjust raw claim counts to a more consistent basis in order to mitigate or eliminate potential inconsistencies. It may also be possible to employ inconsistent claim counts in a manner that would minimize the impact of potential distortions.

Unpaid claim counts may be estimated by various actuarial claim count methods including application of the Model. Unpaid claim counts together with an unpaid severity trend are used to estimate r_i as:

Estimated $r_i = \frac{\text{Estimated } C_{i,d}}{\text{Estimated } C_{i-1,d-1}} \text{ x } \frac{\text{Estimated } S_{i,d}}{\text{Estimated } S_{i-1,d-1}}$,

where $C_{i,j}$ = accident year *i* number of claims unpaid as of year-end *j,* where $j \geq i$,

$S_{i,j}$ = accident year *i* unpaid severity as of year-end *j,* where $j \geq i$.

The entire quantity $\frac{\text{Estimated } S_{i,d}}{\text{Estimated } S_{i-1,d-1}}$ may be estimated as the estimated unpaid severity percent increase of accident year *i* losses as of accounting date year-end *d* over estimated unpaid severity for accident year *i*-1 losses as of accounting date year-end *d*-1. For example, unpaid severity of accident year *i* losses as of accounting date year-end *d* estimated to be 3% greater than estimated unpaid severity of accident year *i*-1 losses as of accounting date year-end *d*-1 corresponds to $\frac{\text{Estimated } S_{i,d}}{Estimated\ S_{i-1,d-1}}$ equals 1.03.

Where data are organized by report year, claim counts are generally known by report year end. As such, the relative ratio of unpaid claims are known. Consequently, only an estimate of unpaid severity trend is required in order to estimate r_i in such a report year setting.

4.7 Other Measures and Adjustments

Depending upon the line of business, it may be worthwhile to investigate exposure measures not previously discussed. These include payroll, number of vehicles, miles driven, operating expenditures, square footage, average occupied beds, outpatient visits, and number of employees. Accident year-over-year comparisons of these types of measures may provide additional insight into appropriate estimated r_i.

It may be appropriate to adjust exposure measures for features that may not otherwise be captured. Adjustments may be appropriate for items such as policy limits and deductibles, reinsurance provisions, law changes, and tabular reserves. Littmann [6] and Struzzieri and Hussian [8] explore exposure adjustment concepts in greater detail. For the purposes of applying the Model, the key question of whether to adjust relative exposure candidate(s) is: Does the proposed adjustment(s) improve the accuracy of estimated r_i?

4.8 Optimal Estimated Relative Unpaid Losses r_i

As evidenced by the foregoing discussion, an optimal measure of r_i estimates cannot be universally prescribed to cover all circumstances. Further investigation may be warranted when competing initial r_i candidates result in divergent r_i indications. Additional insight may also be gained by exploring the sensitivity of unpaid claim estimates to several reasonable r_i indications. Within a business segment, it may be plausible that appropriate exposure measures may vary by accident year. It may also be reasonable to use a weighted average of different potential r_i measures as an appropriate r_i measure. The key principle is that optimal estimated r_i is the relative exposure measure (or combination of exposure measures) that most accurately estimates the ratio of exposure to unpaid losses for accident year *i* as of accounting date year-end *d* relative to unpaid losses for accident year *i*-1 as of accounting date year-end *d*-1.

Where indicated r_i have low credibility, it may be advisable to restrict the number of successive accident years included in the application of the Model. For example, relatively small remaining unpaid claim exposure for the oldest several accident years may result in volatile low credibility r_i indications for these accident years. It may be prudent to exclude these accident years, especially to the extent that low credibility r_i would have a leveraged effect on the unpaid loss indications for subsequent accident years. An extreme example would be a relatively old accident year with no remaining unpaid claims liability that results in an undefined or indeterminate (division by zero) r_i indication. This issue is discussed further in Section 5.

5. OLDEST ACCIDENT YEAR UNPAID LOSSES $U_{m,d}$

Successful implementation of the Model requires a reasonable estimate of unpaid losses (including IBNR) for the oldest included accident year, $U_{m,d}$. Examples in this paper have accepted the company filed loss reserves (including IBNR) for the oldest accident year as the corresponding unpaid losses. Estimating unpaid losses for the oldest accident year is akin to estimating the tail in traditional loss development methods. The CAS Committee on Reserves [1] has compiled an extensive set of techniques to estimate tail factors. Many of these techniques may be readily adapted to estimate unpaid losses for the oldest included accident year.

Each application of the Model requires one to consider the oldest accident year *m* to include in the calculation. Under optimal circumstances: *m* is set at the oldest accident year with unpaid claim exposure as of accounting date year-end *d*; $U_{m,d}$ and each r_i are credible; and relatively small changes in $U_{m,d}$ and r_i result in relatively small changes in the resulting unpaid claims estimate. Where these conditions are not met, it may be more appropriate to set *m* equal to a later year than the oldest accident year in order to more closely approximate optimal Model conditions. Unpaid losses for accident years prior to *m* would normally be expected to be relatively small and may be estimated by methods other than applying the Model.

6. EMPIRICAL EVIDENCE

The CAS loss reserve data base [7][11] can be used to empirically compare the relative accuracy of commonly used loss reserving methods versus methods derived from the Model. Although the goal of the CAS data base is to "prepare a clean and nice data set of loss triangles that could be used for claims reserving studies," several issues preclude the use of every included company business segment for unbiased comparison (e.g., data abnormalities, sparseness). Consequently, each business segment is pre-screened for inclusion in the comparisons. For the 46 business segments that meet qualifying criteria, Table 6 uses actual emergence as a retrospective test to compare accuracy of 12/31/97 unpaid loss estimates for (a) the Payment Development Method, (b) the Incurred Development Method, (c) the Bornhuetter-Ferguson Method, and four (4) relative unpaid claims methods (*d*)-(*g*) derived from application of the Model.

TABLE 6

RETROSPECTIVE ACCURACY TEST OF 12/31/97 UNPAID CLAIM ESTIMATES:
46 Qualifying CAS Loss Reserve Data Base U.S. Property/Casualty Business Segments

(1) Loss Reserving Method	(2) Number of Business Segments where Estimate Falls Within 20% of Actual Emergence	(3) Number of Business Segments where Estimate Falls Within 10% of Actual Emergence
Payment Development (a)	19	13
Incurred Development (b)	26	17
Bornhuetter-Ferguson (c)	32	21
Relative Unpaid Claims 1 (d)	30	16
Relative Unpaid Claims 2 (e)	27	18
Relative Unpaid Claims 3 (f)	38	21
Relative Unpaid Claims 4 (g)	33	23

(2) Number of 46 Business Segments where $1/1.2 \leq$ (Estimated Unpaid Loss)/(Actual Emergence) ≤ 1.2
(3) Number of 46 Business Segments where $1/1.1 \leq$ (Estimated Unpaid Loss)/(Actual Emergence) ≤ 1.1

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[11] The CAS data base is "a data set that contains [net of reinsurance] run-off triangles of six lines of business [private passenger auto liability/medical; commercial auto/truck liability/medical; workers' compensation; medical malpractice – claims made; other liability – occurrence; and products liability] for all U.S. property casualty insurers. The triangle data correspond to claims of accident year 1988 -1997 with 10 years of development lag. Both upper and lower triangles are included so that one could use the data to develop a model and then test its performance retrospectively". The Section 3 example uses data from a large business segment (Company Code 1767) of other liability experience drawn from the CAS data base.

6.1 Criteria for Inclusion

Business segments were pre-selected from the CAS data base for consistency, credibility and compatibility with each of the seven (7) methods under consideration. Recalling that all dollar figures presented throughout this paper are displayed with thousands of dollars omitted, each selected business segment must meet the following criteria:

- Actual emergence of at least $25,000;
- Positive earned premium for each calendar year 1988 through 1997;
- Non-negative calendar year 1997 loss payments for each accident year 1988 through 1997;
- Each accident year 1988 through 1996 case reserve as of 12/31/96 at least equal to $25 and each accident year 1989 through 1997 case reserve as of 12/31/97 at least equal to $25; and
- No division by zero in working through any of the seven methods.

This filtering results in 46 business segments for comparison testing including the Section 3 example business segment.

6.2 Seven Unpaid Claim Methods

Ordinarily, sound actuarial practice would not blindly rely upon mechanical 'cookbook' procedures. Nevertheless, in order to objectively analyze and compare method performance, it is necessary to make standardized assumptions. If the only information available were the CAS loss reserve data base experience as of accounting date 12/31/97, we attempt to standardize how a practicing actuary might typically implement three commonly applied loss reserving methods – Payment Development Method, Incurred Development Method, and Bornhuetter-Ferguson Method. Four methods derived from the Model are also standardized.

All seven methods accept accident year 1988 company filed loss reserves (including IBNR) as of accounting date 12/31/97 as the estimate for the corresponding unpaid losses. For the calculation of CDFs, it follows that this filed loss reserve plus accident year 1988 cumulative paid losses through 12/31/97 are assumed to be accident year 1988 ultimate losses. The 10 year-ultimate tail payment (or reported) development CDF is, therefore, assumed to equal these accident year 1988 ultimate losses divided by accident year 1988 cumulative loss payments (or reported losses) through 12/31/97.

The seven standardized methods used to estimate 12/31/97 accounting date unpaid losses are discussed as follows:

Payment Development Method[12] (a)- For each development period, select each LDF equal to the (up to) three most recent dollar weighted average payment LDFs as of accounting date 12/31/97.

Incurred Development Method[13] (b)- For each development period, select each LDF equal to the (up to) three most recent dollar weighted average reported LDFs as of accounting date 12/31/97.

Bornhuetter-Ferguson Method[14] (c)- Select Expected Loss Ratio equal to combined accident years 1988 through 1990 Incurred Development Method estimated ultimate loss ratio[15]. For accident years where Incurred Development method CDF>1.000, select these CDFs for use in the Bornhuetter-Ferguson Method. For accident years where Incurred Development method CDF ≤ 1.000, select accident year Incurred Development Method estimated ultimate losses.

Relative Unpaid Claims Method 1 (*d*)- Assume r_i equals case reserve ratios as implemented in Section 3.

Relative Unpaid Claims Method 2 (e)- Assume r_i equals estimated one year reported emergence ratios as implemented in Section 4.3.

Relative Unpaid Claims Method 3 (f)- Assume r_i equals 0.75 x (r_i of Relative Unpaid Claims Method 1) + 0.25 x (earned premium ratios as in Table 4.5, Column (3)). Assigning 75% weight to case reserve ratios and 25% weight to earned premium ratios is one approach to estimating r_i by blending a loss experience-based estimate with an *a priori* earned premium based estimate.

Relative Unpaid Claims Method 4 (*g*)- Assume r_i equals 0.75 x (r_i of Relative Unpaid Claims Method 2) + 0.25 x (earned premium ratios as in Table 4.5, Column (3)).

Since the CAS data base does not capture claim count experience, it does not permit us to also explore and compare unpaid claim estimates using reserving methods that rely upon claim counts.

6.3 Accuracy Measure

Unpaid loss estimates are calculated using all seven Section 6.2 methods for each of the 46 qualifying business segments. Table 6 is a retrospective accuracy test that displays the number of business segments where the 12/31/97 estimated unpaid claim estimate fall within 20% and

12 Friedland [4], Chapter 7

13 Ibid

14 Ibid, Chapter 9, Incurred Bornhuetter-Ferguson version

15 Selection of an Expected Loss Ratio is, perhaps, the most challenging assumption to standardize. Other possibilities were considered such as: choosing a different number of years than the oldest three accident years- however, three years seems to strike a reasonable balance between capturing loss ratio information and not simply reiterating Incurred Loss Development method indications; using the company incurred losses (including IBNR) for more than the oldest accident year in the numerator of the expected loss ratio calculation instead of ultimate losses indications from the Incurred Loss Development method- however, this would incorporate company knowledge absent from the other six methods; choosing a fixed expected loss ratio (e.g., 60%, 65%) across all accident years for all business segments- however, this would ignore the loss ratio tendencies of the particular business segment; and choosing expected loss ratios in conjunction with the historical industry underwriting cycle- however, this would use information external to the CAS loss reserve data base unlike any of the other six methods.

10%[16] of actual emergence. Notwithstanding randomness, methods where more of the 46 business segments have unpaid claim estimates that fall within a specified range are empirically more accurate than those methods where fewer fall within that range.

6.4 Discussion of Results

Based upon review of Table 6, we observe the empirical comparative accuracy of the seven loss reserving methods tested.

The relatively poor performance of the Payment Development Method is consistent with Forray's [3] observation that this method should not generally receive the weight it often does. The Incurred Development Method is best compared with Relative Unpaid Claims Method 1 and Relative Unpaid Claims Method 2 since these all only rely upon payments and case reserves (or estimated one year reported emergence) without reference to earned premium exposure. Although requiring much less historical experience, Relative Unpaid Claims Method 1 performs similarly to the Incurred Development Method. Relative Unpaid Claims Method 2 slightly outperforms the Incurred Development Method.

The Bornhuetter-Ferguson Method outperforms the other two traditional reserving methods. This is also consistent with Forray's [3] inference that the incurred Bornhuetter-Ferguson Method is the best performing method in common use[17]. The Bornhuetter-Ferguson Method is most comparable to Relative Unpaid Claims Methods 3 and 4 since these all consider earned premium exposure. Unlike the Bornhuetter-Ferguson Method, Relative Unpaid Claims Methods 3 and 4 have the significant advantage that selection of expected loss ratios is not required. By assigning one-quarter weight to earned premium ratios, we are attempting to bring stability and additional credibility to estimated r_i. Relative Unpaid Claims Methods 3 and 4 perform at least as well as the Bornhuetter-Ferguson Method. The best performing method for the 20% range is Relative Unpaid Claims Method 3 and the best performing method for the 10% range is Relative Unpaid Claims Method 4.

While Relative Unpaid Claims Methods 3 and 4 use one particular weighting scheme (75% weight to case reserve, or estimated one year reported emergence, ratios; 25% weight to earned premium ratios) to estimate r_i, many other weightings between case reserve (or estimated one year reported emergence) ratios and earned premium ratios may also be reasonable. One possibility is to

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16 Since the distribution of liability unpaid losses is typically right skewed:
Actual emergence within 20% of estimate is defined as $1/1.2 \leq$ (estimated unpaid loss)/(actual emergence) ≤ 1.2;
Actual emergence within 10% of estimate is defined as $1/1.1 \leq$ (estimated unpaid loss)/(actual emergence) ≤ 1.1.

17 Forray measures comparative performance via relative "Method Skill". The expected loss ratios used in Forray's incurred Bornhuetter-Ferguson formulation are industry-based.

formulate a credibility weighting scheme between case reserve (or estimated one year reported emergence) ratios and earned premium ratios. Another avenue for exploration is to incorporate r_i steady state properties into a credibility weighting procedure. Investigation of suitable credibility weightings is a fertile area for future research.

No attempt is made to apply rigorous statistical tests of significance to our observations regarding unpaid claims estimates derived from the Model compared with traditional actuarial loss reserving methods. However, our heuristic approach generally suggests that unpaid claim estimates derived from applications of the Model are at least as accurate as comparable unpaid loss estimates derived from commonly applied actuarial loss reserving methods. In any case, perceived overall improved accuracy over a specific historical data set would not guarantee improved accuracy for any particular future instance where the Model may be applied.

7. SUMMARY RESULTS AND DISCUSSION

This paper presents a straightforward Relative Unpaid Claims Loss Reserving Model. Examples are presented to highlight practical applications of the Model and considerations are explored to offer guidance in the selection of appropriate parameters for methods that apply the Model. In general, methods that apply the Model require less data and information and fewer assumptions than traditional chain ladder loss development methods. Empirical testing suggests that unpaid claim estimates derived from applications of the Model are generally as accurate, if not more accurate, than comparable unpaid claim estimates derived from commonly applied actuarial loss reserving methods. In consideration of the above, the loss reserving paradigm set forth in this paper provides a very practical and powerful tool for the estimation of unpaid claims.

8. CONCLUSION AND FUTURE RESEARCH

With its focus on appropriate parameters that measure prospective emergence, the Relative Unpaid Claims Loss Reserving Model provides actuaries the opportunity and flexibility to tailor methods to the circumstances of business segments under review and to directly estimate unpaid losses. While the paper explores many Model parameter options, additional research is encouraged to study techniques to further improve parameter accuracy and, thereby, increase the accuracy of resultant unpaid claims estimates. Other research topics include: rigorous statistical tests comparing the accuracy of Relative Unpaid Claims Loss Reserving versus basic loss reserving methods; special considerations for small books of business and low credibility data; and appropriate treatment of negative loss payments. Although this paper introduces Relative Unpaid Claims Loss Reserving and has concentrated on unpaid claims point estimates, it also paves the way toward future work that would cast the Relative Unpaid Claims Loss Reserving Model in a stochastic framework.

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Abbreviations and notations

ALAE, allocated loss adjustment expenses
CAS, Casualty Actuarial Society
CDF, cumulative loss development factor
DCCE, defense and cost containment expenses
IBNR, incurred but not reported loss (i.e., all unreported development beyond case reserves)
LDF, age-to-age loss development factor
NAIC, National Association of Insurance Commissioners

$ dollars are displayed with thousands of dollars omitted throughout this paper.

"Actual emergence" is defined throughout this paper as cumulative losses paid subsequent to 12/31/97 through nine years subsequent to the accident year plus company filed loss reserves (including IBNR) as of nine years subsequent to the accident year.

Biography of Author

Bertram A. Horowitz is President of Bertram Horowitz, Inc. Actuarial and Risk Consultants which provides property/casualty and title insurance actuarial and risk assessment services. He has a B.S. degree in Applied Mathematics from the State University of New York at Stony Brook and a M.S. in Mathematics from Brown University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. Mr. Horowitz is the former Special Deputy Superintendent and Financial Actuary of the New York State Insurance Department (now the New York State Department of Financial Services). He has served on the CAS Committee on Reserves and has been an active participant in the development of actuarial research, principles and standards.

Mr. Horowitz may be contacted at bert@bertramhorowitz.com; Bertram Horowitz, Inc.

APPENDIX A

Closed-Form Model Representation:

$$U_{i,d} = U_{m,d} \prod_{m+1 \le j \le i} r_j + \sum_{m \le k \le i-1} [p_{k,d}] \prod_{k+1 \le j \le i} r_j \text{, where } m+1 \le i \le n$$

Proof:

$$U_{m+2,d} = r_{m+2}[r_{m+1}(U_{m,d} + p_{m,d}) + p_{m+1,d}$$

$$U_{m+3,d} = r_{m+3}\{r_{m+2}[r_{m+1}(U_{m,d} + p_{m,d}) + p_{m+1,d}] + p_{m+2,d}\}$$

$$= r_{m+3}r_{m+2}r_{m+1}(U_{m,d} + p_{m,d}) + r_{m+3}r_{m+2}p_{m+1,d} + r_{m+3}p_{m+2,d}$$

$$U_{i,d} = U_{m,d} \prod_{m+1 \le j \le i} r_j + \sum_{m \le k \le i-1} [p_{k,d}] \prod_{k+1 \le j \le i} r_j$$

Q.E.D.

APPENDIX B, SHEET 1

SECTION 3 EXAMPLE BUSINESS SEGMENT
HISTORICAL INCREMENTAL PAID LOSSES AND CASE RESERVES

Accident Year	Paid During Year 1	*Case Reserves as of Year 1*	Paid During Year 2	*Case Reserves as of Year 2*	Paid During Year 3	*Case Reserves as of Year 3*	Paid During Year 4	*Case Reserves as of Year 4*	Paid During Year 5	*Case Reserves as of Year 5*	Paid During Year 6	*Case Reserves as of Year 6*	Paid During Year 7	*Case Reserves as of Year 7*	Paid During Year 8	*Case Reserves as of Year 8*	Paid During Year 9	*Case Reserves as of Year 9*	Paid During Years 10	*Case Reserves as of Year 10*
1988	3,962	*18,455*	22,325	*32,519*	26,713	*24,536*	39,637	*10,366*	13,699	*6,640*	10,341	*3,393*	3,939	*4,025*	3,161	*3,177*	2,079	*1,588*	2,064	*116*
1989	6,066	*18,674*	22,231	*27,084*	31,838	*16,408*	23,890	*13,583*	19,061	*10,691*	14,446	*6,809*	4,848	*3,791*	3,734	*2,838*	5,085	*1,419*		
1990	3,751	*15,681*	27,752	*32,388*	36,613	*26,127*	32,308	*19,254*	12,364	*12,150*	10,289	*6,913*	6,004	*4,883*	3,432	*1,436*				
1991	3,336	*22,485*	42,852	*38,265*	49,612	*40,475*	39,363	*24,041*	17,983	*16,674*	12,284	*7,016*	13,032	*3,282*						
1992	6,647	*31,730*	42,672	*48,726*	45,541	*43,345*	34,446	*25,248*	18,929	*23,466*	17,241	*11,991*								
1993	8,056	*44,945*	73,031	*69,391*	66,645	*48,541*	45,543	*31,248*	23,924	*15,482*										
1994	9,720	*41,128*	55,619	*62,428*	64,964	*56,994*	56,447	*46,505*												
1995	7,171	*51,969*	75,651	*66,826*	77,480	*55,399*														
1996	16,696	*54,941*	72,104	*70,761*																
1997	21,098	*61,839*																		

CAS Loss Reserve Data Base [7]

APPENDIX B, SHEET 2

SECTION 3 EXAMPLE BUSINESS SEGMENT
HISTORICAL ONE YEAR REPORTED EMERGENCE

Accident Year		*Case Reserves as of Year 1*	**One Year Reported as of Year 2**	*Case Reserves as of Year 2*	**One Year Reported as of Year 3**	*Case Reserves as of Year 3*	**One Year Reported as of Year 4**	*Case Reserves as of Year 4*	**One Year Reported as of Year 5**	*Case Reserves as of Year 5*	**One Year Reported as of Year 6**	*Case Reserves as of Year 6*	**One Year Reported as of Year 7**	*Case Reserves as of Year 7*	**One Year Reported as of Year 8**	*Case Reserves as of Year 8*	**One Year Reported as of Year 9**	*Case Reserves as of Year 9*	**One Year Reported as of Year 10**
1988		18,455	54,844	32,519	51,249	24,536	50,003	10,366	20,339	6,640	13,734	3,393	7,964	4,025	6,338	3,177	3,667	1,588	2,180
	1 Year LDF		2.972		1.576		2.038		1.962		2.068		2.347		1.575		1.154		1.373
1989		18,674	49,315	27,084	48,246	16,408	37,473	13,583	29,752	10,691	21,255	6,809	8,639	3,791	6,572	2,838	6,504	1,419	
	1 Year LDF		2.641		1.781		2.284		2.190		1.988		1.269		1.734		2.292		
1990		15,681	60,140	32,388	62,740	26,127	51,562	19,254	24,514	12,150	17,202	6,913	10,887	4,883	4,868	1,436			
	1 Year LDF		3.835		1.937		1.974		1.273		1.416		1.575		0.997				
1991		22,485	81,117	38,265	90,087	40,475	63,404	24,041	34,657	16,674	19,300	7,016	16,314	3,282					
	1 Year LDF		3.608		2.354		1.566		1.442		1.157		2.325						
1992		31,730	91,398	48,726	88,886	43,345	59,694	25,248	42,395	23,466	29,232	11,991							
	1 Year LDF		2.880		1.824		1.377		1.679		1.246								
1993		44,945	142,422	69,391	115,186	48,541	76,791	31,248	39,406	15,482									
	1 Year LDF		3.169		1.660		1.582		1.261										
1994		41,128	118,047	62,428	121,958	56,994	102,952	46,505											
	1 Year LDF		2.870		1.954		1.806												
1995		51,969	142,477	66,826	132,879	55,399													
	1 Year LDF		2.742		1.988														
1996		54,941	142,865	70,761															
	1 Year LDF		2.600																
1997		61,839 `																	
W'td Avg. Dev. Factor			2.7249017		1.8627350		1.6082550		1.4460186		1.2571046		1.7282284		1.3999528		1.6909393		1.3727960

One Year Reported as of Year x = Appendix B, Sheet 1: Paid During Year x + Case Reserves as of Year x
Wt'd Avg. Dev. Factor equals dollar weighted average of (up to 3) most recent years underlined 1 Year LDFs