Exam 8

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Canadian Institute of Actuaries / Institut canadien des actuaires

October 24, 2018

Exam 8

Advanced Ratemaking

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 52 point examination consists of 17 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid/tape.

   - Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. For your Candidate ID number, four boxes are provided corresponding to one box for each digit in your Candidate ID number. If your Candidate ID number is fewer than 4 digits, begin in the first box and do not include leading zeroes. Your name, or any other identifying mark, must not appear.
   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
   - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have received the reference materials:

   a. National Council on Compensation Insurance, Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2018 Study Kit*).
   b. Insurance Services Office, Inc., Commercial General Liability Experience and Schedule Rating Plan (*Excerpt from 2018 Study Kit*)
   c. National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2018 Study Kit*).

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

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9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 7, 2018.

END OF INSTRUCTIONS

1. (9 points)

An insurance company is planning to expand into a new territory and has decided to review its historical loss experience in order to determine whether it will require additional capital to support the expansion.

The insurance company has engaged an actuarial consultant to provide insights into a prospective loss ratio for the new territory. The following table outlines the insurance company's historical experience for two long-tailed lines of business (LOB):

Accident Years	Earned Premiums		Ultimate Losses		Ultimate Claim Counts	
	LOB 1	LOB 2	LOB 1	LOB 2	LOB 1	LOB 2
1991-1995	$12,033,000	$1,766,000	$2,329,000	$1,236,000	170	170
1996-2000	$13,812,000	$1,819,000	$2,762,000	$1,273,000	210	172
2001-2005	$13,985,000	$1,751,000	$2,797,000	$1,506,000	210	201
2006-2010	$16,444,000	$1,710,000	$3,288,000	$1,471,000	240	195
2011-2015	$17,507,000	$1,673,000	$3,350,000	$1,439,000	250	198
Total	$73,781,000	$8,719,000	$14,526,000	$6,925,000	1,080	936

a. (1.5 points)

Conduct chi-square tests with an α value of 0.10 on actual vs. expected claim counts to confirm whether or not risk parameters have shifted over time. Use the following table of critical values:

Degrees of Freedom	Critical Value $\alpha = 0.10$
1	2.706
2	4.605
3	6.251
4	7.779
5	9.236
6	10.645

b. (1.5 points)

To select an expected future claim frequency for LOB 2, the actuarial consultant has decided to assign equal weight (Z/2) to each of the most recent two groups of accident years and the remaining weight (1-Z) to the overall mean frequency.

Calculate the expected future claim frequency for LOB 2 by first using the mean-squared-error (MSE) criterion to determine the optimal value for Z from the following three choices:

Z value	MSE
0.10	Not provided
0.50	0.0190%
0.90	0.0164%

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(The following information relates to parts c., d., e., f., and g. below)

The insurance company is planning to write \$10,000,000 of new business for LOB 1 in the new territory. The following reinsurance treaty options are available to the insurance company to support its expansion:

Option 1: 2,000,000 xs 6,000,000 Aggregate Excess of Loss
- Rate on Line = 12.5%

Option 2: 25% Aggregate Quota Share
- Ceding Commission = 20%
- Target ceded profit of 20%, which specifies that if ceded premiums less ceding commission and ceded loss exceeds 20% of ceded premiums, the excess is paid back to the cedent through a profit commission
- Aggregate Ceded Loss Ratio Cap = 220%

c. (2.25 points)

Calculate the ceded profit for each reinsurance option for the following gross loss amount scenarios:

Scenario	LOB 1 Gross Loss Amount
1	5,000,000
2	10,000,000
3	20,000,000
4	30,000,000

d. (1.5 points)

The actuarial consultant has decided that a prospective loss ratio of 20% for LOB 1 is appropriate, but that the gross capital required to support the expansion is \$8,000,000 based on the insurance company's requirement to hold capital at a 1-in-100 year return period. Assume that aggregate LOB 1 ground-up losses are lognormally distributed.

The cumulative distribution function of a lognormal distribution is: $F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$, where Φ is the standard normal CDF, and μ and σ are the mean and standard deviation of the associated normal distribution.

The mean and variance of a lognormal distribution are given below, as well as a table of values from the standard normal distribution function:

$$E(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

$$Var(X) = [\exp(\sigma^2) - 1]\exp(2\mu + \sigma^2)$$

x	Φ(x)
1.751	96.00%
1.814	96.52%
1.881	97.00%
1.916	97.23%
2.054	98.00%
2.121	98.31%
2.326	99.00%
2.531	99.43%

Determine the coefficient of variation for LOB 1, assuming it is less than 100%.

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e. (1 point)

Calculate the probability of each of the following for LOB 1 assuming aggregate losses are lognormally distributed:

i. Attaching the 2,000,000 xs 6,000,000 Aggregate Excess of Loss treaty
ii. A negative ceded profit on the Aggregate Quota Share

f. (0.5 point)

State one advantage and one disadvantage of the Aggregate Quote Share from the perspective of the reinsurer.

g. (0.75 point)

Recommend and justify which reinsurance treaty option the insurance company should select for LOB 1. Assume both reinsurance treaty options provide the same amount of capital relief.

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2. (9.5 points)

A certain golf course holds a series of tournaments throughout the year, during which prizes are awarded if a player hits a hole-in-one. The prize amounts vary based on the difficulty of the hole, but are consistent across tournaments.

The course has approached an insurer to design a policy which would protect the course from large prize payouts. The insurer would pay the full prize amount directly to the winning player, and seek recoveries of any applicable deductible from the golf course.

A single annual policy will cover all of the tournaments throughout a given year.

The total number of holes-in-one in a given year (n), across all tournaments, has the following probability distribution:

n	$P(n)$
0	0.80
1	0.15
2	0.05
3 or more	0.00

Given that a hole-in-one has occurred, the prize amount (x) has the following probability distribution:

x	$P(x)$
$100,000	0.40
$250,000	0.35
$500,000	0.24
$1,000,000	0.01

The expenses and profit for this policy are:

Fixed Expenses		None
General Expenses	8%	of premium
Commission	10%	of premium
Taxes	5%	of premium
Underwriting Profit	5%	of premium

The above assumptions apply to all program structures.

a. (1 point)

Calculate the annual guaranteed cost premium for this policy.

b. (1 point)

Calculate the annual premium for this policy under a $100,000 per-occurrence deductible.

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c. (1.75 points)

Calculate the annual premium for this policy under a $1,000,000 annual aggregate deductible and no per-occurrence deductible.

d. (1 point)

Calculate the annual premium for this policy under a $250,000 franchise deductible.

e. (1 point)

Explain which of the deductible structures above would generate the greatest credit risk to the insurer, and estimate the magnitude of that risk.

f. (1.25 points)

The insurer has allocated $45,000 of capital to support the guaranteed cost policy in part a. above.

Recommend and justify which of the three options below would be the most appropriate capital requirement to support the per-occurrence deductible policy in part b. above:

i. $45,000
ii. $32,000
iii. $19,000

g. (2.5 points)

Based on the success of this program, the insurer wishes to expand this coverage to other golf courses.

The insurer will offer varying per occurrence limits with no aggregate limits, the prize amounts would remain the same, and the same distribution assumptions would apply.

Using $100,000 as a basic limit, use the variance method to calculate the risk-loaded ILF for a $500,000 policy limit. Assume $\rho(100{,}000) = 5{,}750$ and use $k = 0.0000005$.

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3. (2.75 points)

An insurance company has a private passenger auto book of business with the following claims experience:

Group	Number of Accident-Free Years	Earned Premiums	Current Merit Rating Factor	Number of Claims Incurred
A	3 or more	216,000,000	0.60	25,000
X	2	135,000,000	0.75	18,000
Y	1	63,750,000	0.85	20,000
B	0	200,000,000	1.00	C
Total		614,750,000		63,000 + C

- Claim counts follow a Poisson distribution with parameter $\lambda = 0.05$.
- The credibility for the new policy period for an insured that has had no claim-free years is equal to 0.038.

a. (1.5 points)

Calculate C, the number of claims incurred for Group B.

b. (0.75 point)

Calculate the merit rating factor for an exposure that is accident-free for two or more years for the new policy period.

c. (0.5 point)

Briefly explain two circumstances under which using earned premium as the exposure base would not correct for maldistribution.

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4. (0.75 point)

An insurance company is launching a new telematics program for their private passenger automobile book of business. Telematics devices record various attributes such as miles driven and braking practices. Management decided to give a 5% discount to all customers that participate in the program. The Department of Insurance questions the filing and wants the company to address the following potential concerns:

- Risk of adverse selection
- Relationship between risk and expected outcomes
- Practicality of monitoring the discount's effectiveness

Defend the use of the discount by briefly addressing each of the concerns in light of Actuarial Standard of Practice No. 12, Risk Classification.

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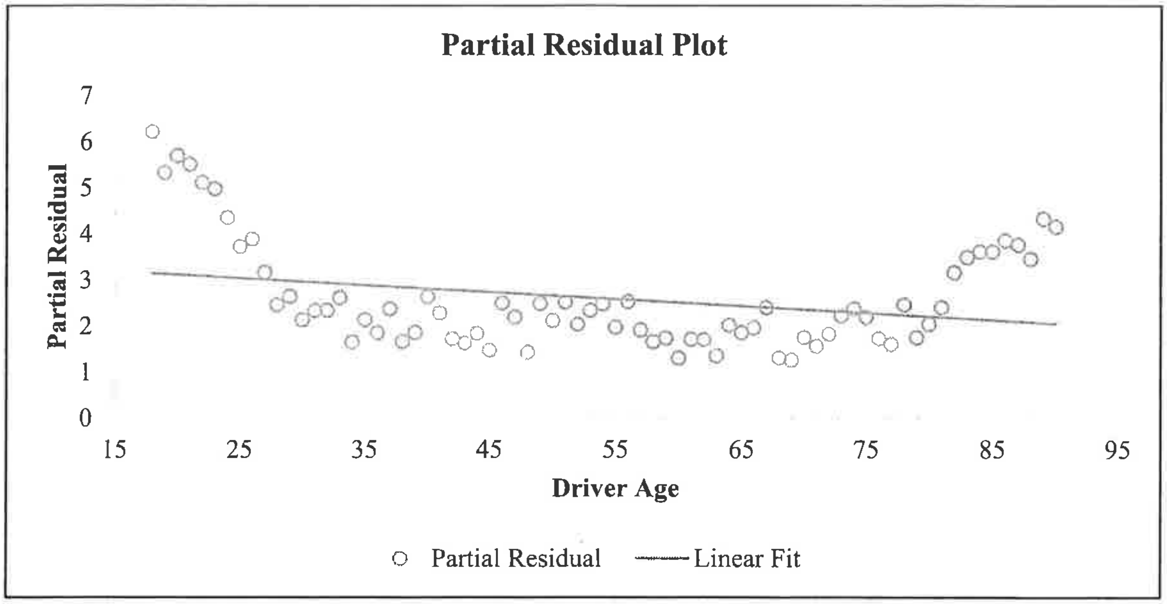
5. (1.5 points)

An actuary is analyzing a partial residual plot of the driver age variable, which is shown below:



a. (1 point)

Adding polynomial terms is one approach to address the non-linearity in the driver age variable.

Briefly describe two other alternative approaches and how they can be used to improve the fit of the driver age variable shown above.

b. (0.5 point)

Briefly describe a downside to each of the two alternative approaches recommended in part a. above.

6. (2.5 points)

An actuary is comparing the output of two generalized linear models to develop a new rating plan for personal auto. Model statistics are shown below:

	Saturated Model	Model A	Model B
Log-Likelihood	-1,000	-1,500	-1,465
Estimated Dispersion Parameter	1.75	1.75	1.75

- Model A is a nested model of Model B, where Model B has an additional variable for driver age.
- Driver age is fit using a second-order polynomial.
- The critical value to be used from the F-distribution is 19.5.

a. (2 points)

Using two statistical tests, recommend whether or not driver age should be included in the rating plan.

b. (0.5 point)

Describe why the deviance statistic alone should not be used to assess model fit.

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7. (2.5 points)

An actuary is planning to add a credit-based insurance score to a model that estimates the probability of a policy having a claim. The actuary has decided to offset all of the current model variables before fitting the new variable.

Given the following:

- The current model (without the insurance score variable) is a logit-link binomial GLM (logistic regression).
- The logit link function is defined as $g(\mu) = \ln\left(\frac{\mu}{1-\mu}\right)$
- The insurance score is a continuous variable having a value between 1 and 100.
- The current fitted values and insurance score for three policies as well as regression results from the fit of the insurance score variable are given below:

Policy Number	Fitted Probability Without Insurance Score	Insurance Score
1	1.3%	78
2	20.3%	92
3	2.5%	35

Variable	Parameter Estimate
Intercept	1.250
Insurance Score	-0.020

a. (0.5 point)

Calculate the offset term to be used in the regression for each of the three policies above.

b. (0.75 point)

Calculate the revised fitted probability of having a claim for each of the three policies above.

c. (0.5 point)

Identify the range of:

i. the logit function
ii. the logistic function

d. (0.25 point)

Briefly explain why logistic regression is often used to model probabilities.

e. (0.5 point)

Identify and briefly describe one situation in which the use of an offset is preferable to (re)fitting all variables.

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8. (1.5 points)

An actuary has been using risk-adjusted increased limit factors to account for riskiness in pricing. The actuary's coworker has suggested an alternate approach of using non-risk-adjusted increased limit factors and, instead, varying the profit and contingency load with the policy limit.

Compare and contrast the two methods with respect to each of the following:

- Accuracy
- Ease of calculation
- Clarity

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9. (4.25 points)

Three insurers are using experience rating to determine premiums for a specific class of business. The same ten risks were rated using the experience plan of each insurer. Information related to each rating plan is given below:

Insurer 1's Plan

Risk #	Manual Premium	Loss	Mod	Standard Premium
1	$810	$750	0.97	$786
2	$900	$490	0.68	$612
3	$950	$1,075	1.13	$1,074
4	$975	$650	0.78	$761
5	$1,075	$850	0.88	$946
6	$1,100	$1,000	0.96	$1,056
7	$1,225	$1,300	1.06	$1,299
8	$1,300	$800	0.72	$936
9	$1,450	$1,175	0.90	$1,305
10	$1,500	$975	0.76	$1,140

Insurer 2's Plan

Quintile	Manual Loss Ratio	Standard Loss Ratio
1	58.6%	94.5%
2	65.7%	90.0%
3	80.2%	85.3%
4	91.6%	79.7%
5	109.2%	75.3%
Sample Variance	0.0411	0.0059

Insurer 3's Plan

Efficiency Test Statistic
0.0000

a. (2.75 points)

Rank each of the insurers' rating plans from most to least equitable using the Efficiency Test as described by Fisher et al.

b. (1.5 points)

Explain how adverse selection may affect each of the three insurers in a well-functioning insurance market. Assume that no adjustments are made to the experience rating plans over time.

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10. (2.25 points)

An actuary is analyzing an experience rating plan using three years of experience that has the following information:

Policy effective date	January 1, 2019
Policy term	One Year
Annual loss trend	2%
Cap for individual claims	100,000
Credibility factor	0.60
Expected ultimate loss before modification	500,000

Reported Losses on Individual Claims by Policy Year as of June 30, 2018			
2015	2016	2017	2018
3,450	2,389	456	5,694
5,000	345	126,890	99,832
234	1,236,806	2,345	76,532
98,000		1,874	
324,789		690	
		26,986	

The actuary has determined limited loss development factors (LDFs) and limited expected values using aggregate data from similar policies as follows:

Maturity to Ultimate	Limited LDFs
18 months	1.50
30 months	1.23
42 months	1.20

Limit	Limited Expected Value
100,000	17,500
500,000	28,567
1,000,000	43,393
Unlimited	85,504

a. (1.25 points)

Calculate the expected reported losses for this plan for the three years of experience combined.

b. (1 point)

Calculate the experience modification factor for this policy.

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11. (1.25 points)

Given the following information for a construction insured's general liability policy that is subject to the ISO Commercial General Liability Experience and Schedule Rating Plan:

Company Subject Loss Cost	270,500
Actual Experience Ratio	0.85
Policy Effective Date	January 1, 2020
Policy Term	Annual

All major changes in the last five years for this insured:

- Upgraded all equipment in 2015
- Improved employee training in 2018
- Implemented a new safety program in 2018

a. (0.5 point)

Calculate the experience modification for this policy.

b. (0.75 point)

An underwriter has selected a credit of 10% for schedule rating. Assess the reasonability of this selection.

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12. (3 points)

An insured is considering an incurred loss retrospective rating plan and a paid loss deductible plan with the following characteristics:

Incurred Loss Retro	
Per Occurrence Limit	\$150,000
Maximum Ratable Loss	\$500,000
Paid Loss Deductible	
Per Occurrence Deductible	\$150,000
Aggregate Deductible	\$500,000
The following apply to both plans:	
Expected Ultimate Losses	\$450,000
Expected Ultimate Losses limited by occurrence	\$350,000
Expected Ultimate Losses limited by occurrence and aggregate	\$325,000
Fixed underwriting expenses including commission & profit	\$100,000
LAE as a percentage of Loss	8.0%
Tax Multiplier	1.030

a. (1.5 points)

Compare the expected total cost of insurance of the two plans, taking into account both premium and deductible payments, from the perspective of the insured.

b. (0.5 point)

Recommend and briefly justify one of the two plans above from the perspective of the insured.

c. (1 point)

Propose two changes to the recommended plan in part b. above and briefly explain why each would be beneficial for the insured.

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13. (2.75 points)

An insurance company observes the following ground-up claim experience for a book of business:

- Claim counts (N) follow a Poisson distribution with $\lambda = 5{,}000$
- Claim size (X) follows a Pareto distribution with $\alpha = 2$ and $\beta = 6{,}000$

The insurance company pays claims in excess of $10,000.

Given the following:

- For a Poisson distribution:

$$\Pr(N = n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

- For a Pareto distribution:

$$f(x) = \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\beta}{x+\beta}\right)^\alpha$$

$$E[X;x] = \frac{\beta}{\alpha-1}\left[1 - \left(\frac{\beta}{x+\beta}\right)^{\alpha-1}\right]$$

a. (0.75 point)

Calculate the expected number of claims in excess of $10,000.

b. (2 points)

Assume claim size is subject to a uniform annual inflation rate of 3%.

Calculate the rate at which ground-up claim counts must change in order for there to be no change in expected annual total aggregate excess losses.

14. (3 points)

The balanced plan provisions for a workers' compensation risk in the state of Alabama are given below:

Standard Premium	\$9,000,000
Minimum Entry Ratio, r_H	0.28
Maximum Entry Ratio, r_G	2.55
Loss Conversion Factor	1.30
Tax Multiplier	1.05
State Hazard Group Differential	0.90
Adjusted Expected Loss	\$4,860,000

- Using the 2008 NCCI table of expected loss ranges, the expected retrospective premium = \$9,493,205.
- There is no per-occurrence loss limit.

a. (1.25 points)

Calculate the basic premium ratio to standard premium.

b. (1.25 points)

The insured believes that the insurance charge embedded in the current basic premium is unfair and cites the following five years of loss experience the insured had with a prior carrier:

Year	Loss Ratio
1	40%
2	165%
3	15%
4	55%
5	25%

The insured's exposure base has remained stable over time.

Compare the net insurance charge in the current basic premium for this policy to the net insurance charge based on the prior loss experience using the plan provisions given above.

c. (0.5 point)

Discuss the appropriateness of using a basic premium derived from the prior loss experience.

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15. (1.75 points)

A reinsurer has priced a quota share treaty to achieve an expected combined ratio of 90%. Expenses for the treaty are as follows:

Ceding Commission	20%
Brokerage Fees	5%
Administrative Expenses	1%
Unallocated Expenses	1%

The following table represents the expected loss ratio distribution for the primary insurer under the treaty:

Range of Loss Ratios	Average Ratio in Range	Probability of being in Range
0-40%	37.3%	0.03
40-60%	53.2%	0.21
60-80%	66.1%	0.55
80% or above	91.1%	0.21

The treaty includes a loss corridor from 60% to 80% loss ratio.

Calculate the percent of loss reassumed by the primary insurer in the loss corridor.

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16. (1.75 points)

A reinsurance company is evaluating whether or not to write a \$50 million excess of \$50 million catastrophe reinsurance contract with a primary insurer. The reinsurer is currently holding \$850 million of capital and is required to hold enough capital to survive a 1-in-250 event. Without the new contract, the reinsurance company has a 1-in-250 probable maximum loss (PML) of \$825 million which is solely driven by the hurricane peril.

Given the following:

- The primary insurer's PMLs are driven by the hurricane and earthquake perils only.
- The primary insurer's aggregate annual PMLs by return period are as follows:

Return Period (years)	PML (\$000,000)
1000	125
500	105
200	95
100	70
50	50
25	30
20	25
10	20
5	15

- The largest hurricane event in the primary insurer's event catalog is \$45,000,000.

a. (1 point)

Calculate the ceded, and net, 1-in-250 PMLs for this contract for the primary insurer.

b. (0.75 point)

Evaluate whether the reinsurer should participate in this treaty.

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17. (2 points)

A catastrophe modeler would like to incorporate a new construction technique into a catastrophe model. This new technique would theoretically reduce the amount of building damage sustained during hurricane force winds. However, experts have not reached a consensus on the effectiveness of the new construction technique because it has not been exposed to an actual hurricane.

a. (0.5 point)

Briefly describe which module(s) of the catastrophe model would need to be modified to account for the new information.

b. (0.5 point)

Classify the uncertainty created with the new construction technique as either aleatory or epistemic and briefly justify the selection.

c. (1 point)

Briefly describe and contrast two methods the modeler could use to incorporate uncertainty in this catastrophe model.

END OF EXAMINATION

Exam 8
Advanced Ratemaking

POINT VALUE OF QUESTIONS

QUESTION	VALUE OF QUESTON	SUB-PART OF QUESTION (a)	(b)	(c)	(d)	(e)	(f)	(g)
1	9.00	1.50	1.50	2.25	1.50	1.00	0.50	0.75
2	9.50	1.00	1.00	1.75	1.00	1.00	1.25	2.50
3	2.75	1.50	0.75	0.50				
4	0.75	0.75						
5	1.50	1.00	0.50					
6	2.50	2.00	0.50					
7	2.50	0.50	0.75	0.50	0.25	0.50		
8	1.50	1.50						
9	4.25	2.75	1.50					
10	2.25	1.25	1.00					
11	1.25	0.50	0.75					
12	3.00	1.50	0.50	1.00				
13	2.75	0.75	2.00					
14	3.00	1.25	1.25	0.50				
15	1.75	1.75						
16	1.75	1.00	0.75					
17	2.00	0.50	0.50	1.00				
18	0.00							
19	0.00							
20	0.00							
21	0.00							
22	0.00							
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39	0.00							
40	0.00							
41	0.00							
42	0.00							
43	0.00							
44	0.00							
45	0.00							
TOTAL	52.00							

SAMPLE ANSWERS AND EXAMINER'S REPORT

FALL 2018 EXAM 8 EXAMINER'S REPORT

The Syllabus and Examination Committee has prepared this Examiner's Report as a tool for candidates preparing to sit for a future offering of this exam. The Examiner's Report provides:

- A summary of exam statistics.
- General observations by the Syllabus and Examination Committee on candidate performance.
- A question-by-question narrative, describing where points were commonly achieved and missed by the candidate.

The report is intended to provide insight into what the graders for each question were looking for in responses that received full or nearly-full credit. This includes an explanation of common mistakes and oversights among candidates. We hope that the report aids candidates in mastering the material covered on the exam by providing valuable insights into the differences between responses that are comprehensive and those that are lacking in some way.

Candidates are encouraged to review the Future Fellows article from June 2013 entitled "Getting the Most out of the Examiner's Report" for additional insights.

EXAM STATISTICS:

- Number of Candidates: 953
- Available Points: 52.00
- Passing Score: 33.75
- Number of Passing Candidates: 314
- Raw Pass Ratio: 32.9%
- Effective Pass Ratio: 35.1%

The Syllabus and Examination Committee understands the pass ratio for this exam is lower than recent prior sittings, and as a result spent additional time analyzing the results prior to selecting the pass mark. In determining the final pass score, the following two additional actions were taken beyond those normally made in the grading process:

- In recognition that the exam may be longer than those from recent prior sittings, an aggregate downward adjustment was made to the pass score, determined based on various metrics.
- An additional review was performed of complete exam papers for candidates whose scores were slightly below the pass score, to gain a better understanding of the appropriateness of the time adjustment and pass score selected. These exam papers were reviewed by a team of volunteers to evaluate the reasons the candidates did not reach the pass score, specifically whether they appeared to be candidates that knew the material well but ran out of time or candidates that lost points due to misunderstanding of concepts.

Based on these additional steps, the Syllabus and Examination Committee is satisfied that the selected passing score is reasonably consistent with the standard that candidates have been held to in the past.

We understand this explanation is of little comfort to those candidates who did not achieve the passing score. We hope that the details by question provided throughout this Examiner's Report will be helpful to those candidates and future candidates. In addition, in an attempt to better assist candidates in preparing for the next sitting of this exam, the Syllabus and Examination Committee notes three specific items that caused a significant number of points to be lost on this exam:

- Candidates are reminded to read the questions carefully. This exam had numerous instances where candidates answered a question other than the one asked, or disregarded important information that was explicitly provided, which led to a loss of credit. This was particularly observed on questions 1 (parts f-g), 4, 6, 9, 10 and 12.

- For a few of the calculation questions on this exam, a short, efficient solution to the question exists, but some candidates answered in a manner that was more complicated and longer than necessary. This not only takes away from time to answer other questions but also increases the likelihood of calculation errors. This was observed mainly on question 2 (parts a-c), but also to a certain extent on questions 3 (part b) and 14 (parts a-b).

- It appears as if some candidates may not have reviewed the Case Study included as part of the Syllabus. Candidates are strongly encouraged to download this Excel file and work through all of the tabs of that file.

GENERAL COMMENTS:

- Candidates should note that the instructions to the exam explicitly say to show all work; graders expect to see enough support on the candidate's answer sheet to follow the calculations performed. While the graders made every attempt to follow calculations that were not well-documented, lack of documentation may result in the deduction of points where the calculations cannot be followed or are not sufficiently supported.
- Integrative Questions (IQs) were first introduced to Exam 8 in 2017 and are being used to test candidates' ability to apply and synthesize multiple advanced ratemaking ideas in addressing complex business problems. Both IQs this sitting were based on real-world scenarios and were designed to test multiple syllabus learning objectives at higher cognitive (Bloom's) levels. Candidates should expect to encounter similar sorts of questions in future sittings.
- Candidates are reminded of the following excerpt from the Exam 8 syllabus: "The ability to apply ratemaking knowledge and experience may be tested through questions dealing with problems for which there are no generally recognized solutions. The readings for Exam 8 should be studied for illustration of basic principles and theories, as well as for insight into advanced ratemaking problems and their solutions." This applies not only to Integrative Questions, but to the entire exam overall.
- Incorrect responses in one part of a question did not preclude candidates from receiving up to full credit for correct work on subsequent parts of the question that depended upon that response. This includes situations where candidates could not calculate an answer but made a reasonable one up in order to make further progress on the later part(s) of the question.

SAMPLE ANSWERS AND EXAMINER'S REPORT

- Candidates should be cognizant of the way an exam question is worded. They must look for key words such as "briefly" or "fully" within the problem. We refer candidates to the Future Fellows article from December 2009 entitled "The Importance of Adverbs" for additional information on this topic.
- Candidates should note that the sample answers provided in the examiner's report are not an exhaustive representation of all responses given credit during grading, but rather the most common correct responses.
- In cases where a given number of items were requested (e.g., "three reasons" or "two scenarios"), the examiner's report often provides more sample answers than the requested number. The additional responses are provided for educational value, and would not have resulted in any additional credit for candidates who provided more than the requested number of responses. Candidates are reminded that, per the instructions to the exam, when a specific number of items is requested, only the items adding up to that number will be graded (i.e., if two items are requested and three are provided, only the first two are graded).
- It should be noted that all exam questions have been written and graded based on information included in materials that have been directly referenced in the official Syllabus, which is located on the CAS website. The CAS takes no responsibility for the content of supplementary study materials and/or manuals produced by outside corporations and/or individuals that are not directly referenced in the official Syllabus.

QUESTION 1	
TOTAL POINT VALUE: 9	**LEARNING OBJECTIVE(S): A1, B1, B2, C3, C4**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

D.O.F = n-1 = 5 – 1 = 4
Critical Value = 7.779

$$\chi^2 = \sum \frac{(A-E)^2}{E}$$

Avg Freq LOB 1: 1080/73781 = 0.0146
Avg Freq LOB 2: 936/8719 = 0.1074

Accident Years	LOB 1 Actual	LOB 1 Expected	LOB 2 Actual	LOB 2 Expected
1991-1995	170	176.14	170	189.58
1996-2000	210	202.18	172	195.27
2001-2005	210	204.71	201	167.97
2006-2010	240	240.71	195	183.57
2011-2015	250	256.27	198	179.60
Chi-Squared		0.809		8.295

LOB 1 H_O: Risk parameters have not shifted over time
0.809 < 7.779
LOB 1 count parameters have not shifted, accept null hypothesis

LOB 2 H_O: Risk parameters have not shifted over time
8.295 > 7.779
Reject null hypothesis, parameters have shifted as the χ^2 statistic is greater than the threshold.

Sample 2:

Since exposure is not given, use prem as exposure base

Accident Years	Freq LOB 1	Freq LOB 2
1991-1995	170/12033 = 0.01413	170/1766 = 0.09626
1996-2000	0.01520	0.09456
2001-2005	0.01502	0.11479
2006-2010	0.01459	0.11404
2011-2015	0.01428	0.11835

Expected freq assumed to be the overall mean: LOB1 -> 1080 / 73781 = 0.01464 LOB2 -> 936 / 8719 = 0.10735 LOB1: $\chi^2 = \sum \frac{w(expected - actual)^2}{expected}$ $\chi^2 = \frac{12033(0.01413-0.01464)^2+\cdots+17507(0.01428-0.01464)^2}{0.01464}$ = 0.8084 < 7.779 Therefore no shift over time LOB2: $\chi^2 = \sum \frac{w(expected - actual)^2}{expected}$ $\chi^2 = \frac{1766(0.09626-0.1074)^2+\cdots+1673(0.1184-0.1074)^2}{0.1074}$ = 8.296 > 7.779 Therefore there is a shift in risk parameters over time.
Part b: 1.5 points
Sample 1 195/1710 = .1140 Freq/1000 prem 2006 – 2010 198/1673 = .1184 Freq/1000 prem 2011 – 2015 936/8719 = .1074 All Years 170/1766 = .0963 (1991-1995) 172/1819 = .0946 (1996-2000) 201/1751 = .1148 (2001-2005) For Z=0.1: (.05)(.0963) + (.05)(.0946) + (.9)$\left(\frac{170+172}{1766+1819}\right)$ = .0954 expected 2001-2005 (.05)(.1148) + (.05)(.0946) + (.9)$\left(\frac{170+172+201}{1766+1819+1751}\right)$ = .1021 expected 2006-2010 (.05)(.1140) + (.05)(.1148) + (.9)$\left(\frac{170+172+201+195}{1766+1819+1751+1710}\right)$ = .1057 expected 2011-2015 $((.1148 - .0954)^2 + (.1140 - .1021)^2 + (.1184 - .1057)^2)/3 = 0.00022642$ =.0226% = MSE for Z = 0.1 MSE is lowest for Z = 0.9, so we use that selection. Expected Freq: (.45)(.1140) + (.45)(.1184) + (.1) (.1074) = .1153

Sample 2

Test Z = 10% - choosing to only test the most recent 3 years groups since they have 2 prior groups each

AY	Actual Freq	Expected Freq w/Z=0.1
01-05	201/1751M=0.000115	$(.05)\left(\frac{170}{1766M}\right)+(.05)\left(\frac{172}{1819M}\right)+(.9)\left(\frac{170+172}{1766M+1819M}\right)$=0.0000954
06-10	0.000114	0.0001021
11-15	0.000118	0.0001057

Overall freq = 936/8719000 = 0.000107

$$MSE = \sum \frac{(Actual - Exp)^2}{3}$$

$= 2.3 \times 10^{-10}$

So we use Z=0.1, since it has the lowest MSE

Then expected frequency is projected = 0.9(0.000107) + 0.05(198/1673000) + 0.05(195/1710000) =0.000108 claims/earned premium

Part c: 2.25 points

Sample 1

Option 1:
Ceded premium = 12.5% × 2,000,000 = $250,000

Scenario	Gross Loss	Ceded Premium	Ceded Loss	Ceded Profit
1	5,000,000	250,000	0	250,000
2	10,000,000	250,000	2,000,000	-1,750,000
3	20,000,000	250,000	2,000,000	-1,750,000
4	30,000,000	250,000	2,000,000	-1,750,000

Option 2:
Ceded Premium = 25% (10,000,000) = 2,500,000
Ceding Commission = 20% (2,500,000) = 500,000

Scenario	Gross Loss	Ceded Loss	Ceded Premium	Ceding Commission	Profit Commission
1	5,000,000	25% (5,000,000) =1,250,000	2,500,000	500,000	250,000
2	10,000,000	2,500,000	2,500,000	500,000	0
3	20,000,000	5,000,000	2,500,000	500,000	0
4	30,000,000	220%(2,500,000) =5,500,000	2,500,000	500,000	0

Scenario	Ceded Profit
1	2,500,000-500,000-250,000-1,250,000 = 500,000
2	2,500,000-500,000-2,500,000 = -500,000
3	2,500,000-500,000-5,000,000 = -3,000,000
4	2,500,000-500,000-5,500,000 = -3,500,000

Sample 2

Scenario 1: Gross loss \$5M
Option 1:
Ceded Premium = 2M × 12.5% = 250,000
Ceded loss = 0 => Profit = 250,000
Option 2:
Ceded premium = 10M × 25% = 2,500,000
Ceded Loss = 5M × 25% = 1,250,000 => Ceded LR = 50%
(Premium – Loss - Commission) / Premium = 1 - 0.5 – 0.2 = 30% > 20%
=> Ceded Profit = 1 - 0.50 – 0.20 – 0.10 = 20%, or 20% × 2.5M = 500,000

Scenario 2: Gross loss \$10M
Option 1: Ceded loss = 2M => Profit = -1.75M
Option 2: Ceded loss = 2. 5M => Ceded LR = 100%
=> Profit = 1-1-.2 = -20%, or -500,000

Scenario 3: Gross loss \$20M
Option 1: Ceded loss = 2M => Profit = -1.75M
Option 2: Ceded loss = 20×.25 = 5M => Ceded LR = 5M/2.5M = 200%
=> Profit = 1-2-.2 = -120%, or -3.0M

Scenario 4: Gross loss \$30M
Option 1: Ceded loss = 2M => Profit = -1.75M
Option 2: Ceded loss = 30×.25 = 7.5M => Ceded LR = 7.5M/2.5M = 300% > Ceded LR Cap
⇨ Ceded LR = 220% => Profit = 1-2.2-.2 = -140%, or -3.5M

Sample 3

Option 1:
Premium = \$2M × 12.5% = 250,000
Scenario 1: Ceded loss = 0 Ceded profit = 100%
Scenario 2: Ceded loss = 2M " " = 1 – 2,000,000/250,000 = -700%
Scenario 3: " " = 2M " " = -700%
Scenario 4: " " = 2M " " = -700%

Option 2:
Ceded Premium = 10M × 0.25 = 2,500,000
Max Ceded = 2.5 × 220% = 5.5M

Scenario 1: Ceded loss = 5M × 0.25 = 1.25M
Ceded profit = min(20%, (2.5 × (1-20%)-1.25)/2.5M) = 20%
Scenario 2: Ceded loss = 10M × .25 = 2.5M
Ceded profit = min(20%, (2.5 × (1-20%)-2.5)/2.5) = -20%
Scenario 3: Ceded loss = 5M
Ceded profit = min(20%, (2.5 × (1-20%)-5)/2.5) = -120%
Scenario 4: Ceded loss = min(5.5,30 × .25) = 5.5M
Ceded profit = min(20%, (2.5 × (1-20%)-5.5)/2.5) = -140%

Part d: 1.5 points

Sample 1

$2{,}000{,}000 = \exp(\mu + \sigma^2/2)$
$[\ln(8{,}000{,}000) - \mu]/\sigma = 2.326$
$15.8950 - 2.326\sigma = \mu$
$2{,}000{,}000 = \exp[15.8950 - 2.326\sigma + \sigma^2/2]$
$-1.3863 = -2.326\sigma + \sigma^2/2$
$-2.7727 = -4.6520\sigma + \sigma^2$
$\sigma^2 - 4.6520\sigma + 2.7727 = 0$
$\sigma = (4.6520 \pm 3.2481)/2$
$\sigma = 0.7019$ or $\cancel{3.9501}$
$\mu = 14.2624$ or $\cancel{6.7071}$
$\text{stddev}(x) = \exp(0.7019^2 - 1)(\exp(2(14.2624) - 0.7019^2)^{1/2} = 1{,}595{,}790$
$CV = 1{,}595{,}790/2{,}000{,}000$
$= 0.7979$.

Sample 2

20% LR → $E[X] = 2M$
1/100 years, 8M : $P(X>8m) = 0.01$ or $P(X<8M) = 0.99$
$(\ln(x) - \mu)/\sigma = 2.326$
$E(X) = \exp(\mu + \sigma^2/2) = 2M$
$\exp(\ln(8M) - 2.326\sigma + \sigma^2/2) = 2M \rightarrow \sigma = 0.7019$
$CV(X) = \text{sqrt}(\exp(\sigma^2)-1)$
$= 0.798$ (since $CV<1$)

Part e: 1 point

Sample 1

i) $P(X>6M) = 1 - F(6M) = 1 - \Phi[(\ln(6M) - 14.3634)/0.7019]$
$= 1 - \Phi(1.916)$
$= 1 - 0.9723 = 0.0277$

ii) Negative ceded profit occurs when $2M - \text{Loss} \times 0.25 < 0$ or $\text{Loss} > 8M$.
We know 8M is 1 in 100 years, so prob. of negative ceded profit on option 2 is 1%.

Sample 2

i) $F(6M) = \Phi[(\ln(6M) - \mu)/\sigma] = \Phi(1.916) = .9723$ (from the table)
$P(X>6M) = 1 - F(6M) = 1 - .9723 = 2.77\%$

ii) Breakeven → Ceded losses = ceded premium & commission

25%X=10M(25%)(1-20%)

.25%X = 2M

X=8M

So ceded profit <0 if X>8M

P(X>8M) = 1/100 years = 0.01.

Part f: 0.5 point

Sample Responses for Advantage of QS

- It has a ceded loss ratio cap (220%) which limits the downside to the reinsurer
- Reinsurer can share in the profits of the primary insurer
- Insurer will have financial incentive for risk control
- Large ceded premium which can be used to earn investment income
- Easier to administer since same % is ceded on every loss
- Less volatility compared to an XS plan as the ceded amount is a fixed % from ground up
- Results should be more stable since the reinsurer assumes a fixed percentage of loss
- Learns about losses on a 1st dollar basis so reinsurer will have better knowledge of what's coming up in terms of development and will have shorter report lag

Sample Responses for Disadvantage of QS

- Large loss potential when loss experience is bad
- Reinsurer has to pay more ceding commissions
- Profit commission limits the reinsurer's upside
- Reinsurer needs to pay ceding commission up-front, may have cash flow disadvantage

Part g: 0.75 point

Sample Responses for choosing Aggregate QS

- The aggregate QS protects the insurer from adverse loss scenarios up to a 220% loss ratio while the aggregate XOL only protects a small portion ($2M) of adverse loss scenarios. The aggregate QS also has profit commission which limits the reinsurer's profit (reward insurer for good experience)
- QS has lower ceded profits and offers more coverage in bad years (when loss > $8M)
- Choose QS: more protection against tail events; profit sharing if good loss experience is realized
- Select the QS, more coverage (ground up coverage at 25%). The $2M limit from the agg XOL is small. Ceded commission seems reasonable. Profit commission in place for favorable loss experience.
- Choose QS since it will provide ceding commission, better cash flow advantage for insurer, can use the money to earn investment income

Sample Responses for choosing Aggregate XOL

- The loss ratio for the insurer's book is very stable. For a more stable and profitable LOB, an aggregate XOL is preferred so that less profit is ceded in good years and tail events can be protected with relatively lower ceded premium

- Recommend option 1 for the insurer, as it is cheaper upfront, providing a cashflow advantage and still protects against risk of potential large losses
- I would choose the aggregate excess of loss. The loss ratio is low so presumably it is profitable. The QS would be ceding profitable business whereas the Agg XOL would only cede in the event losses get too high

EXAMINER'S REPORT

Candidates were expected to test for shifting parameters for each LOB and make a projection of claim frequency for one in particular.

Candidates were then expected to analyze two reinsurance options under multiple loss scenarios for the other LOB, and evaluate both options from the perspectives of the primary insurer and the reinsurer.

Part a

Candidates were expected to understand how to test for shifting risk parameters by performing chi-squared tests on expected claim count. They were expected to test the lines separately and appropriately use earned premium as an exposure base, accounting for growth and changes in the mix of business.

To receive full credit, candidates needed to use earned premium as the exposure base, as it was the only option available.

Common mistakes included:

- Not using earned premium as an exposure base and ignoring growth when calculating expected claims
- Combining the two lines of business before testing and ignoring mix of business changes
- Selecting the wrong degrees of freedom to determine the critical value

Part b

Candidates were expected to calculate MSE for Z=0.1, determine the optimal Z value using the MSE criterion, and calculate the future frequency estimate. To do this, candidates needed to calculate what the frequency estimates would have been historically (using the cumulative frequency available up to that point in time) for Z=0.1.

The MSE values given for Z=0.5 and Z=0.9 in the question were calculated using $1000s of premium as the exposure base, but this was not identified in the question. Because candidates were not expected to recalculate these values, those that calculated frequencies in dollars of earned premium rather than in $1000s also received full credit, assuming the correct conclusions were reached using these values (see Sample 2).

Common mistakes included:

- No attempt to calculate MSE for Z=0.1 and at the same time not providing any justification for the same
- Incorrect formula for MSE
- Using the same estimated frequency for each of the prior years in the MSE calculation

- Calculating the estimated frequencies using the overall five-year frequency as the credibility complement instead of the cumulative frequency to date
- Selecting the correct Z value but neglecting to calculate a point estimate for future expected claim frequency
- Performing calculations using claims counts instead of frequencies

Part c

Candidates were expected to calculate ceded profit as defined in the question itself for each loss scenario separately. For aggregate excess of loss treaty (Option 1), the ceded profit is equal to ceded premium less ceded loss (as there was no ceding commission). For aggregate quota share treaty (Option 2), the ceded profit is equal to ceded premium less ceded losses and ceding commission. The candidate was expected to make correct adjustments for the profit commission for Scenario 1 and the aggregate ceded loss ratio cap for Scenario 4. The profit could be stated in dollars or as a percentage of ceded premium.

Some candidates averaged or aggregated the scenarios, possibly interpreting the four scenarios as individual claims or losses. Because no weights were given, this interpretation is not reasonable. However, such solutions received partial credit if candidates showed sufficient work to demonstrate their understanding of the different treaty terms used in the question.

Other common mistakes included:

- Mixing up limit and retention for excess of loss treaty, or applying rate on line to gross premium instead of the treaty limit
- Calculating target profit commission and/or loss ratio cap by applying given percentages to ceded premium net of ceding commission, or using different quota share percentages (e.g. 20% quota share)
- Mixing up ceded profit with profit commission or not recognizing negative profits

Part d

Candidates were expected to understand aggregate loss distributions with respect to the relationship between μ, σ, Var(X) and E(X), and various reinsurance structures.

Candidates were expected to understand the proper relationships in order to set up a system of equations for F(X) and E(X), and then solve for μ and σ. Candidates were then expected to calculate the coefficient of variation by either solving for Var(X) and E(X) or by recognizing that the coefficient of variation simplifies (for a lognormal random variable) to $\sqrt{e^{\sigma^2} - 1}$.

Common mistakes included:

- Using historical losses or the scenarios from part c to solve for μ and σ
- Using historical losses or the scenarios from part c to solve for Var(X) and E(X)
- Setting μ as 2,000,000
- Forgetting to take the inverse of Φ to convert 0.99 to 2.326
- Taking the inverse of Φ of 0.98 instead of 0.99
- Calculating the coefficient of variation as σ/μ

- Calculating the coefficient of variation as E(X)/sd(X)
- Using a value other than 8,000,000 as the required capital. The question specifically stated that "the Gross Capital required to support the expansion is $8,000,000"

Part e

Candidates were expected to understand the proper amount at which the aggregate excess of loss treaty attaches (that is, at the attachment point) and to understand the relationship to determine when a negative ceded profit occurs on the aggregate quota share.

Common mistakes included:

- Not recognizing that the question asked for the probability that the loss is greater than the threshold. That is, supplying *F*(6mil) instead of *P*(X>6mil) = 1 – *F*(6mil)
- Calculating the probability of attachment as *F*(8mil)-*F*(6mil) or $\frac{F(8mil)-F(6mil)}{1-F(6mil)}$
- Setting up the ceded profit equation incorrectly

Part f

Candidates were expected to understand types of reinsurance contracts and common provisions in reinsurance contracts. Both responses that either related directly to the specific treaties in the question, or the general characteristics of quota share reinsurance were accepted.

Common mistakes included:

- Providing advantages/disadvantages from the perspective of the primary insurer instead of the reinsurer
- Providing a response that is too vague (e.g. easy to calculate, simple to administer), unless the candidate offered appropriate justification for that answer (e.g. simpler to administer because only summary-level data is needed)
- Stating that an advantage from the reinsurer's perspective is receiving more premium without mentioning the timing of premium (cashflow benefits) or recognizing that the reinsurer is also taking on more risk
- Only providing an advantage and not providing a disadvantage (or vice versa)
- Providing two advantages or disadvantages instead of one of each (only the first provided is graded).

Part g

Candidates were expected to compare the two reinsurance options presented in the problem, make a recommendation for which option the insurer should select, and give reasonable justifications for the selection.

Common mistakes included:

- Not providing justification for the selection or providing too vague of an explanation
- Providing justifications that do not match the recommendations
- Making a recommendation from the reinsurer's perspective
- Misunderstanding aggregate excess of loss policy as per occurrence or per risk excess of loss policy
- Not understanding that the primary insurer would prefer the option with the lower ceded

profit (calculated in part c)

QUESTION 2	
TOTAL POINT VALUE: 9.5	**LEARNING OBJECTIVE(S): B1, B2, B5, B6, B7**
SAMPLE ANSWERS	
Part a: 1 point	

Sample 1

E[N] = 0×0.8+1×0.15+2×0.05 = 0.25
E[X] = 100K×0.4+250K×0.35+500K×0.24+1M×0.01 = 257,500
E[S] = 0.25×257,500 = 64,375
Expense Multiplier = 1/(1-0.08-0.1-0.05-0.05) = 1.38889
GC Premium = 64,375×1.38889 = 89,410

Sample 2

E[N] = 0×0.8+1×0.15+2×0.05 = 0.25
E[X] = 100K×0.4+250K×0.35+500K×0.24+1M×0.01 = 257,500
E[S] = 0.25×257,500 = 64,375
GC Premium = 64,375/(1-0.08-0.1-0.05-0.05) = 89,410

Sample 3

Loss 1	Loss 2	Total Loss	prob(n)	prob (loss1)	prob (loss 2)	prob(n)× prob(loss1)× prob(loss2)	Total Loss × total prob
0		0	0.8			0.8	0
100		100	0.15	0.4		0.06	6
250		250	0.15	0.35		0.0525	13.125
500		500	0.15	0.24		0.036	18
1,000		1,000	0.15	0.01		0.0015	1.5
100	100	200	0.05	0.4	0.4	0.008	1.6
100	250	350	0.05	0.4	0.35	0.007	2.45
100	500	600	0.05	0.4	0.24	0.0048	2.88
100	1,000	1,100	0.05	0.4	0.01	0.0002	0.22
250	100	350	0.05	0.35	0.4	0.007	2.45
250	250	500	0.05	0.35	0.35	0.006125	3.0625
250	500	750	0.05	0.35	0.24	0.0042	3.15
250	1,000	1,250	0.05	0.35	0.01	0.000175	0.21875
500	100	600	0.05	0.24	0.4	0.0048	2.88
500	250	750	0.05	0.24	0.35	0.0042	3.15
500	500	1,000	0.05	0.24	0.24	0.00288	2.88
500	1,000	1,500	0.05	0.24	0.01	0.00012	0.18
1,000	100	1,100	0.05	0.01	0.4	0.0002	0.22
1,000	250	1,250	0.05	0.01	0.35	0.000175	0.21875
1,000	500	1,500	0.05	0.01	0.24	0.00012	0.18
1,000	1,000	2,000	0.05	0.01	0.01	0.000005	0.01

The sum of the Total Loss × total prob = 64.375K
GC Premium = 64.375/(1-.08-.05-.05-.1) =89.410K

Sample 4

Loss Amount	Prob
0	0.8
100	=.15×.4 = .06
200	=.05×.4^2= .008
250	=.15×.35 = .0525
350	=.05×.4×.35×2 = .014
500	=.15×.35 + .05×.35^2 = .042125
600	=.05×.24×0.4×2 = .0096
750	=.05×.35×0.24×2=.0084
1,000	=.15×.01+.05×.24^2 = .00438
1,100	=.05×.01×.4×2 = .0004
1,250	=.05×.01×.35×2 = .00035
1,500	=.05×.01×.24×2 = .00024
2,000	=.05×.01^2 = .000005

=0×.8 + 100×.06 + 200×.008 + …. + 2,000×.000005 = 64,375
GC Premium = 64,375/(1-0.08-0.1-0.05-0.05) = 89,410

Part b: 1 point

Sample 1

x	*P(x)*
0	0.40
150,000	0.35
400,000	0.24
900,000	0.01

E[N] = 0.25
E[X] = 150K×0.35+400K×0.24+900K×0.01 = 157,500
E[S] = 0.25×157,500 = 39,375
Deductible Premium = 39,375×1.38889 = 54,688

Sample 2

Holes	Prob	Deductible
0	0.8	0
1	0.15	100,000
2	0.05	200,000

100K×.15 + 200K×.05 = 25,000
64,375-25,000 = 39,375
39,375/.72 = 54,688

Sample 3

LER = 100,000/257,500 = 0.388349
64,375×(1-.388349) = 39,375
39,375/.72 = 54,688

Sample 4

Loss 1	Loss 2	Total Loss	prob(n)	prob (loss1)	prob (loss 2)	prob(n)× prob(loss1)× prob(loss2)	Total Loss × total prob
0		0	0.8			0.8	0
0		0	0.15	0.4		0.06	0
150		150	0.15	0.35		0.0525	7.875
400		400	0.15	0.24		0.036	14.4
900		900	0.15	0.01		0.0015	1.35
0	0	0	0.05	0.4	0.4	0.008	0
0	150	150	0.05	0.4	0.35	0.007	1.05
0	400	400	0.05	0.4	0.24	0.0048	1.92
0	900	900	0.05	0.4	0.01	0.0002	0.18
150	0	150	0.05	0.35	0.4	0.007	1.05
150	150	300	0.05	0.35	0.35	0.006125	1.8375
150	400	550	0.05	0.35	0.24	0.0042	2.31
150	900	1,050	0.05	0.35	0.01	0.000175	0.18375
400	0	400	0.05	0.24	0.4	0.0048	1.92
400	150	550	0.05	0.24	0.35	0.0042	2.31
400	400	800	0.05	0.24	0.24	0.00288	2.304
400	900	1,300	0.05	0.24	0.01	0.00012	0.156
900	0	900	0.05	0.01	0.4	0.0002	0.18
900	150	1,050	0.05	0.01	0.35	0.000175	0.18375
900	400	1,300	0.05	0.01	0.24	0.00012	0.156
900	900	1,800	0.05	0.01	0.01	0.000005	0.009

The sum of the Total Loss × total prob = 39.375K

Deductible Premium = 39.375/(1-.08-.05-.05-.1) =54.688K

Sample 5

Loss Amount	Prob
0	0.8
150	=.15×.35 + .05×.4×.35×2 = .0665
300	=.05×.35^2 = .006125
400	=.15×.24 + .05×.4×.24×2 = .0456
550	=.05×.35×0.24×2=.0084
800	=.05×.24^2 = .00288
900	=.15×.01 + .05×.01×.4×2 = .0019
1,050	=.05×.01×.35×2 = .00035
1,300	=.05×.01×.24×2 = .00024
1,800	=.05×.01^2 = .000005

=0×.8 + 150×.0665 + 300×.006125 + …. + 1,800×.000005 = 39,375

Deductible Premium = 39,375×1.3888889= 54,688

Part c: 1.75 points

Sample 1

In order to exceed the aggregate deductible, must have 2 claims and at least 1 needs to be $1M.

2nd Claim	Probability	Claim × Prob
100,000	2×.05×.01×.4 = .0004	40
250,000	2×.05×.01×.35 .00035	87.5
500,000	2×.05×.01×.24 = .00024	120
1,000,000	.05×.01×.01 = .000005	5

40 + 87.5 + 120 + 5 = 252.5

252.5/.72 = 350.7

Sample 2

Agg Loss	Prob	Agg Loss × Prob
0	0.8	0
100	.15×.4 = 0.06	6
200	.05×.4×.4 = 0.008	1.6
250	.15×.35 = 0.0525	13.125
350	2×.05×.35 = 0.014	4.9
500	.15×.24 + .05×.35×.35 = 0.042125	21.0625
600	2×.05×.4×.24 = 0.0096	5.76
750	2×.05×.35×.24 = 0.0084	6.3
1000	1-sum of above = 0.005375	5.375

6+1.6+13.125+4.9+21.0625+5.76+6.3+5.375 = 64.1225

64.375-64.1225 = .2525

.2525×1000/.72 = 350.7

Sample 3

Loss 1	Loss 2	Total Capped at $1M	prob(n)	prob (loss1)	prob (loss 2)	prob(n)× prob(loss1)× prob(loss2)	Total Loss × total prob
0		0	0.8			0.8	0
100		100	0.15	0.4		0.06	6
250		250	0.15	0.35		0.0525	13.125
500		500	0.15	0.24		0.036	18
1,000		1,000	0.15	0.01		0.0015	1.5
100	100	200	0.05	0.4	0.4	0.008	1.6
100	250	350	0.05	0.4	0.35	0.007	2.45
100	500	600	0.05	0.4	0.24	0.0048	2.88
100	1,000	1,000	0.05	0.4	0.01	0.0002	0.2
250	100	350	0.05	0.35	0.4	0.007	2.45
250	250	500	0.05	0.35	0.35	0.006125	3.0625
250	500	750	0.05	0.35	0.24	0.0042	3.15
250	1,000	1,000	0.05	0.35	0.01	0.000175	0.175
500	100	600	0.05	0.24	0.4	0.0048	2.88
500	250	750	0.05	0.24	0.35	0.0042	3.15
500	500	1,000	0.05	0.24	0.24	0.00288	2.88
500	1,000	1,000	0.05	0.24	0.01	0.00012	0.12
1,000	100	1,000	0.05	0.01	0.4	0.0002	0.2
1,000	250	1,000	0.05	0.01	0.35	0.000175	0.175
1,000	500	1,000	0.05	0.01	0.24	0.00012	0.12
1,000	1,000	1,000	0.05	0.01	0.01	0.000005	0.005

Sum of Total Loss × Total Prob = 64.1225
64.375-64.1225 = .2525
.2525×1000/.72 = 350.7

Part d: 1 point

Sample 1
Eliminate losses 250K and under, pay full amount for claims greater than $250K
500K×.4 + 1M×.01 = 130K
130K×.25 = 32,500
32,500/.72 = 45,139

Sample 2
E[X:250K] = 100K×.4 + 250K×.35 + 250K×.24 + 250K×.01 = 190,000
F(250,000) = .75
LER = (190,000 – 250,000×(1-.75)) / 257,500 = .4951
89,410×(1-.4951) = 45,139

Part e: 1 point

Sample 1
The $1M aggregate deductible generates the most credit risk as the insurer will make all payouts

and then collect from the insured payouts at or below the deductible.

The magnitude of the credit risk is the amount of expected losses below the deductible, which equals \$64,375 – \$252.5 = \$64,122.5.

Sample 2
The insurer pays the prize amount immediately and seeks reimbursement from the insured.

Under the deductible options, the maximum credit risk is:
- 100K Deductible: 2 × 100,000 = \$200,000 (2 hole-in-ones)
- 1M Aggregate: \$1,000,000 (1 or 2 hole-in-ones totaling at least \$1M)
- 250K Franchise Deductible: 2 × 250,000 = \$500,000 (2 hole-in-ones of \$250,000)

Therefore, the \$1M aggregate deductible generates the most credit risk.

Part f: 1.25 points

Sample 1
The needed capital under the deductible will be less than that needed under the GC policy, so eliminate the \$45k option.

From part a, E(S) = 64,375
From part b, E(S) = 39,375

45,000 × 39,375/64375 = 27,524

The deductible covers excess losses which are more volatile, so the capital required should be greater than \$27.5k. Therefore \$19k is too low; select \$32k.

Sample 2
$Var(S) = E(N) \times Var(X) + Var(N) \times E(X)^2$

Under the GC policy, $Var(S) = 2.6455 \times 10^{10}$
Under the deductible option, $Var(S) = 1.4524 \times 10^{10}$

The ratio of the standard deviations is then 1.35
45000/1.35 = 33,333 so select a capital level of \$32k

Sample 3
Return on Capital = Profit/Capital

Under the GC Policy, ROC = 5% × 89410 / 45000 = 9.93%

To keep this same return of 9.93% based on premium of 54688 from part b, the needed capital is 27,520. Accounting for the addition of credit risk, select the \$32k option.

Part g: 2.5 points

Sample 1

$\rho(l) = k[E(X^2{:}l) + \delta\times E(X{:}l)^2]$

$E[X{:}100{,}000] = 100{,}000$

$E[X{:}100{,}000^2] = 100{,}000^2$

$5750 = 0.0000005(100000^2 + \delta\times 100000^2) \rightarrow \delta = 0.15$

$E[X{:}\ 500{,}000] = .4\times 100{,}000+.35\times 250{,}000+.25\times 500{,}000 = 252{,}500$

$E[X^2{:}500{,}000] = .4\times 100{,}000^2 + .35\times 250{,}000^2 + .25\times 500{,}000^2 = 8.8375\times 10^{10}$

$\rho(500{,}000) = 0.0000005\times(8.8375\times 10^{10} + 0.15\times 252{,}500^2) = 48{,}969$

ILF = [252,500 + 48,969)/(100,000+5,750) = **2.851**

Sample 2

$\rho(l) = k[E(X^2{:}l) + \delta\times E(X{:}l)^2]$

$\delta = Var(N)/E(N) - 1$

$E(N) = 1\times.15+2\times.05 = .25$

$Var(N) = (0^2)\times.8+(1^2)\times(.15)+(2^2)\times.05 - (0.25^2) = .2875$

$\delta = .2875/.25 - 1 = .15$

$\rho(500K) = k[E(X^2{:}500K) + \delta E(X{:}500K)^2]$

$E(X^2{:}500K) = 100K^2\times.4 + \ldots\ldots 500K^2\times(.24+.01) = 8.8375\times 10^{10}$

$\rho(500K) = k[8.8375\times 10^{10} + 0.15\times 252{,}500^2] = 48{,}969$

$I(500K) = [E(X{:}500K) + \rho(500K)]/[E(X{:}100K) + \rho(100K)] = (252{,}500 + 48{,}969)/(100{,}000 + 5{,}750)$

I(500K) = **2.851**

Sample 3

$ILF = [E(X{:}500K) + \rho(500K)]/[E(X{:}100K) + \rho(100K)]$

Assuming frequency doesn't change by limit, so only looking @severity.

E(X:100K) = 100,000 (all losses are above 100K or equal)

$E(X{:}500K) = .4(100K) + .35(250K) + .25(500K) = 252{,}500$

$\rho(100K) = 5{,}750$

Assume expenses don't vary by limit

$V(N) = E(N^2) - E(N)^2 = [.15(1^2) + .05(2^2)] - [.25]^2 = .2875$

$V(S) = E(N)V(X) + \{[E(X)]^2\times V(N)\} = (.25)\times[(.4)(100K^2)+(.35)(250K^2)+(.25)(500K^2) - (252{,}500)^2] +$
$(252{,}500^2)(.2875) = 2.448\times 10^{10}$

$\rho(500K) = .0000005[V(S)/E(N)] = 48{,}969$

ILF = (252,500 + 48,969) / (100,000 + 5,750) = **2.85**

EXAMINER'S REPORT

Candidates were expected to be able to calculate annual premium with several different features, including guaranteed cost premium, premium with a per-occurrence deductible, premium with an aggregate deductible, and premium with a franchise deductible. Candidates were then expected to quantify and discuss the relative credit risk of these deductible options and compare capital allocated to guaranteed cost vs. per occurrence deductible policies. Finally, candidates were expected to calculate a risk-adjusted ILF at a given limit.

Part a

Candidates were expected to understand the concepts of frequency, severity and expense multipliers to calculate guaranteed cost premium.

Several candidates developed a discrete distribution of aggregate losses, mapping out each possible permutation. While this is a valid approach, it would have been very time-consuming and would have unnecessarily complicated the solution. Candidates who took this approach had notably more calculation errors in their solutions.

Common mistakes included:

- When approaching the question using the full aggregate distribution: ,
  - Missing a combination of losses when n=2
  - Forgetting to multiply some loss combinations by 2. For example, if Loss 1 = 250 and Loss 2 = 500, the probability of that combination ought to be multiplied by 2 because you could also have Loss 1 = 500 and Loss 2 = 250.
- Using the wrong frequency (used 0.2 instead of 0.25)
- Incorrectly treating part of the expenses as fixed

Part b

Candidates were expected to demonstrate how a per-occurrence deductible works, correctly calculate a new severity, apply the correct frequency and correct expense multiplier to determine the deductible premium.

Common mistakes included:

- When approaching the question using the full aggregate distribution:
  - Missing a combination of losses when n=2
  - Forgetting to multiply some loss combinations by 2. For example, if Loss 1 = 250 and Loss 2 = 500, the probability of that combination ought to be multiplied by 2 because you could also have Loss 1 = 500 and Loss 2 = 250.
- Calculating the premium for the loss under the deductible instead of the excess layer

Part c

Candidates were expected to recognize that in order to reach the aggregate deductible, there must be two claims and at least one of them has to be $1M. The candidate needed to determine the correct probability distribution, apply it to the losses and apply the expense multiplier to arrive at the correct annual premium.

Full credit was not awarded if the candidate treated the $1M as an aggregate limit rather than an aggregate deductible (the problem stated that $1M was the aggregate deductible).
Credit was not awarded to candidates who treated the deductible as a per-occurrence deductible.

Common mistakes included:

- Not including all possible loss combinations (i.e. only including possible aggregates of $2M and $1.5M, but omitting the $1.1M and $1.25M aggregate combinations)
- For candidates who calculated the aggregate loss distribution, missing some combinations due to the volume of possibilities

- Not multiplying the aggregate losses of \$1.1M, \$1.25M and \$1.5M by 2
- Not multiplying by the correct frequency
- Treating the problem like an aggregate limit rather than an aggregate deductible
- Not subtracting the aggregate deductible of \$1M from the loss distribution

Part d

Candidates were expected to demonstrate how a franchise deductible works by applying the correct loss distribution to determine severity, multiply by the correct frequency and expense multiplier to arrive at the correct annual premium. Full credit was not given if a \$250K loss was treated as if there were no deductible and paid on a ground up basis.

Common mistakes included:

- Not including all possible loss combinations
- Not multiplying loss combinations by 2 where Loss 1 does not equal Loss 2
- Assuming that losses of \$250K should be paid in full (from Bahnemann, pg. 178: "The franchise deductible eliminates all claims less than or equal to the deductible or 'franchise' amount *d*, and claims in excess of *d* are paid in full.")
- Treating the franchise deductible as an aggregate deductible rather than a per occurrence deductible

Part e

Candidates were expected to consider the credit risk of the three deductible options presented and opine on the option with the largest credit risk. Additionally, candidates were expected to quantify the amount of credit risk.

Most candidates were successful in identifying that credit risk arises from the insurer seeking reimbursement from the insured for coverage of payouts below the deductible. Considering the expected losses below the various deductible options or the maximum amount of reimbursable losses were both acceptable ways to quantify the magnitude of the credit risk.

Common mistakes included:

- Incorrectly interpreting the coverage structure of the deductible options
- Failing to quantify the magnitude of the credit risk

Part f

Candidates were expected to select and justify, from the list of given options, a capital amount the insurer could hold to support the \$100k per-occurrence deductible.

Almost all candidates recognized that the deductible reduces the needed capital as compared to the guaranteed cost policy.

Candidates took a variety of approaches to quantify the reduction in capital. Comparing the reduction in premium or the reduction in losses from the application of the deductible were most common. Other acceptable responses included relating the reduction in capital to the standard deviations of the two policies, using a PML-based approach to justify the capital selection, or contemplating a return on capital to support the selection.

Finally, to receive full credit candidates were expected to recognize that because the insurer is providing coverage for the more volatile excess layer, the reduction in capital is less than the amount of losses eliminated by the application of the deductible. Stating the introduction of credit risk reduces the overall reduction in capital was also accepted.

Common mistakes included:

- Stating that the capital required should be equal to the expected losses
- Not recognizing that the volatility of excess losses and/or the addition of credit risk causes the reduction in capital to be less than the reduction of primary losses

Part g

Candidates were expected to calculate the risk-loaded ILF at the given increased limit, which involved calculating delta, limited expected severities, and risk loads.

Common mistakes included:

- Using limited loss (frequency × severity) instead of just limited severity
- Using an incorrect formula for the risk load (such as not multiplying the inner terms by k or not squaring E(X:l) in the second term)
- Calculating E(X:l), $E(X^2:l)$ or ρ(l) incorrectly

QUESTION 3	
TOTAL POINT VALUE: 2.75	**LEARNING OBJECTIVE(S): A1, A2**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

$Mod = ZR + (1 - Z)$

$R = \frac{1}{1 - e^{-\lambda}} = \frac{1}{1 - e^{-0.05}} = 20.504$

$Z = 0.038$

$Mod = 0.038(20.504) + (1 - 0.038) = 1.7411$

$Assume\ earned\ premium\ not\ at\ present\ B\ rates.\ \ Adjusted\ earned\ premiums:$

$A - \frac{216{,}000{,}000}{0.6} = 360{,}000{,}000$

$Y - 75{,}000{,}000$

$X - 180{,}000{,}000$

$B - 200{,}000{,}000$

$Total - 815{,}000{,}000$

$$\frac{\frac{C}{200{,}000{,}000}}{\frac{63{,}000 + C}{815{,}000{,}000}} = 1.7411$$

$$\frac{C}{200{,}000{,}000} = \frac{109{,}689.3 + 1.7411C}{815{,}000{,}000}$$

$2.19 * \ 10^{13} + 348{,}220{,}000C = 815{,}000{,}000$

$C = 47{,}000$

Sample 2

$Z_B = 0.038$

$Mod_B = Z * R + (1 - Z)$

$$Approximate\ R\ because\ we\ don't\ have\ prior\ yr\ data \rightarrow\ est.R\ using \frac{1}{1 - e^{-\lambda}} where\ \lambda = 0.05\ so\ R = \frac{1}{1 - e^{-0.05}} = 20.504$$

$$\rightarrow Mod_B = \left(\frac{\frac{C}{200M}}{\frac{63000 + C}{815M}} \right) = 0.038 * 20.504 + (1 - 0.038) = 1.741$$

	EP @ B Rates	# Claims
A	216M/0.6 = 360M	25,000
X	135M/0.75 = 180M	18,000
Y	63.75M/0.85 = 75M	20,000
B	200M	46,948
Total	815M	109,948

$C = 0.427(63000 + C)$

$0.573C = 29{,}601$

$C = 46{,}948$

Part b: 0.75 point

Sample 1

$$Rel\ Freq = Mod$$

$$A + X{:}\ \frac{\dfrac{25{,}000 + 18{,}000}{360{,}000{,}000 + 180{,}000{,}000}}{\dfrac{110{,}000}{815{,}000{,}000}} = 0.59$$

$Merit\ Factor\ is\ 0.59$

Sample 2

$$Mod\ for\ A + X = \left(\frac{\dfrac{25{,}000 + 18{,}000}{360M + 180M}}{\dfrac{109{,}948}{815M}}\right) = 0.590$$

Sample 3

$$Mod\ for\ A + X = \left(\frac{\dfrac{25{,}000 + 18{,}000}{360{,}000 + 180{,}000}}{\dfrac{109{,}948}{815{,}000}}\right) = 0.590$$

$$Mod\ for\ B = 1.741$$

$$Merit\ Factor = \frac{0.59}{1.741} = 0.339$$

Sample 4

$$Merit\ Rating\ = \left(\frac{\dfrac{25 + 18}{360 + 180}}{\dfrac{47}{200}}\right) = 0.3388$$

Part c: 0.5 point

Sample 1

- When high claim frequency territories are not high average premium territories
- When territorial differentials are not proper

Sample 2

- If high frequency (to earned car years) territories were not also high avg premium territories
- If territorial rate differentials were not proper (LRs across territories after applying rate differentials were not approximately equal)

EXAMINER'S REPORT

Candidates were expected to back out the appropriate claim count using the table, credibility, and lambda values provided, calculate the mod for other groups, and understand the proper use of earned premium as an exposure base.

SAMPLE ANSWERS AND EXAMINER'S REPORT

Part a

Candidates were expected to use the provided lambda value to derive R, utilize the mod formula to calculate the mod, and equate that answer to the relative frequency of the correct group. Candidates were also expected to back out the merit rating factor to obtain earned premium at Group B rates.

Common mistakes included:

- Assuming earned premium was already at "B" rates. This was incorrect as it was not explicitly stated as such; rather, the current merit rating factors were provided. The purpose was to back out the current merit rating program before evaluating the credibility of the loss experience provided.
- Setting up the calculation based off the relative frequency of the wrong group (instead of group B)

Part b

Candidates were expected to utilize the claim counts calculated in part a to calculate the merit rating factor for another risk group.

Candidates who calculated the mod (relative frequency to total) or the merit rating factor (relative frequency to group B) received full credit. Candidates who did additional calculations, such as finding Z, were given full credit as long as the mod was correctly calculated and utilized.

Common mistakes included:

- Mislabeling the mod as Z
- Calculating the mod for the incorrect group

Part c

Candidates were expected to know the circumstances under which earned premium was an appropriate exposure base (and corrected for maldistribution).

Most candidates mentioned territory in their responses, but this was not necessary to obtain full credit because maldistribution can be applied to other rating variables as well.

Common mistakes included:

- Stating that loss ratios across territory were not equal, without mentioning that territory differentials were improper (since this alone would not prove territory caused the mispricing)
- Stating circumstances where earned premium is (as opposed to is not) an appropriate exposure base without also stating that this was the case

QUESTION 4	
TOTAL POINT VALUE: 0.75	LEARNING OBJECTIVE(S): A1
SAMPLE ANSWERS	

Sample 1

- Risk of adverse selection:
  - Drivers who know that they drive poorly are unlikely to submit to monitoring.
- Relationship between risk and expected outcomes:
  - Drivers are more likely to drive safely if they know they are being monitored. Therefore, drivers with the discount have a lower expected loss cost.
- Practicality of monitoring the discount's effectiveness:
  - Adoption rates and the experience of non-adopters vs. adopters can be analyzed over time.

Sample 2

- Risk of adverse selection:
  - Adverse selection is not a concern as users of telematics are not likely to be higher cost customers due to improved driving habits.
- Relationship between risk and expected outcomes:
  - The discount is justified as expected outcomes of telematics users is lower costs due to the improved driving habits.
- Practicality of monitoring the discount's effectiveness:
  - The discount is practical to monitor as the company can create a telematics class in their GLM.

Sample 3

- Risk of adverse selection:
  - Risky drivers are less likely to purchase telematics devices since their fast driving / quick braking practices will be recorded. This will attract less risky drivers.
- Relationship between risk and expected outcomes:
  - It is likely that opting for the telematics program will have a correlation with loss experience since less risky drivers will opt for the program (so giving a discount will relate to loss experience).
- Practicality of monitoring the discount's effectiveness:
  - It should be easy to monitor drivers that have the device and measure their loss experience vs. those that don't to see if the discount is appropriate.

Sample 4

- Risk of adverse selection:
  - Worse drivers will not want to be monitored so will either pay the extra 5% or go somewhere else. Drivers control whether they get discount.
- Relationship between risk and expected outcomes:
  - Causality, drivers that know they're being monitored will drive more cautiously → fewer accidents → lower losses
- Practicality of monitoring the discount's effectiveness:
  - Easy to collect loss data by group since discount will be listed in policy in-force as

Y/N then just need to compare loss ratios as they become available. This is being done by other auto companies (used in industry).

Sample 5

- Risk of adverse selection:
  - Only good drivers will opt into this program, so it should help avoid adverse selection and warrant the discount.
- Relationship between risk and expected outcomes:
  - There is an obvious connection between miles driven, braking practices, and losses. We'd expect someone who opts into this program to have fewer miles driven plus better braking, leading to fewer losses.
- Practicality of monitoring the discount's effectiveness:
  - The device will record data so it will be possible to see if drivers opting in are actually better drivers plus have fewer losses, deserving of a discount.

Sample 6

- Risk of adverse selection:
  - Customers who drive infrequently and/or do not brake harshly would seek out a telematics device, assuming future lower premium for them. Therefore, there is potential for favorable selection.
- Relationship between risk and expected outcomes:
  - Given the above, the better risks will have a higher propensity to participate, meaning we would expect the participants to have lower expected losses.
- Practicality of monitoring the discount's effectiveness:
  - As data are collected under the new program, and risks are flagged as participants, we can monitor the effectiveness and reasonability of the 5% magnitude.

Sample 7

- Risk of adverse selection:
  - There shouldn't be risk of adverse selection. Drivers willing to get device are probably lower risk.
- Relationship to loss:
  - Drivers will probably drive more cautiously with device. Should have a relationship to discount.
- Practicality:
  - Should be easy to group risks by insureds with/without device.

Sample 8

- Risk of adverse selection:
  - Insurer can avoid adverse selection since telematics would enable greater risk equity.
- Relationship between risk and expected outcomes:
  - Use industry research / expert opinion to prove that loss cost varies by telematics.
- Practicality of monitoring the discount's effectiveness:
  - I assume insurer has prepared databases correctly, should be practical to compare

loss ratios between those using devices vs. those not using devices and adjust accordingly going forward.

EXAMINER'S REPORT

Candidates were expected to defend the use of the 5% telematics discount by addressing each of the concerns from the Department of Insurance. Many candidates answered the question without relating their responses to the discount given to all customers that participate in the program.

Common mistakes included:

- Discussing the appropriateness of using the attributes recorded by the telematics device (miles driven and braking practices) rather than defending the 5% discount given to all participants
- Arguing against the use of the discount instead of defending
- Stating that the discount was supported by Actuarial Standard of Practice No. 12 but not providing justification
- Explaining concepts discussed in Actuarial Standard of Practice No. 12 but not relating them back to the 5% discount
- Providing justification around the costs and benefits of implementing the telematics program rather than justifying the 5% discount
- Stating that the company would be adversely selected against if they did not implement the discount(while there is a possibility of adverse selection for both adoption and non-adoption of the discount, candidates needed to directly address the selection risk of adopting the discount, regardless of how widespread UBI might be in the marketplace)
- Discussing the long-term benefits of implementing telematics rather than the appropriateness of the discount, which is applied as soon as the policy enrolls in the program
- Stating that the 5% discount could be used to incentivize policyholders to join the program and that this will be recouped once the company starts rating on telematics attributes(while possibly true, this did not directly address any of the criteria in the question)

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 5	
TOTAL POINT VALUE: 1.5	**LEARNING OBJECTIVE(S): A3**
SAMPLE ANSWERS	

Part a: 1 point

Sample 1

- We could bin the variable of drive age, create categorical variables. For this example, binning 16-25, 26-75, 76-96 together would cause the residuals to be more in line with the linear fit.
- We could add a piecewise linear function with hinge points at 25 & 75. This would transform the variables to fit the linear fit.

Sample 2

- Binning the age:
  - bin every 5 years into a group
  - can help differentiate the difference in residuals by age
- Piecewise terms
  - Put into 3 pieces of linear curves
  - "15-25", "25-80", "80+"
  - Can track the different slopes of residuals

Sample 3

- Binning: we can bucket the variables into groups, turn the continuous variable into categorical variables. Each group has a predicted rate & residuals would/should improve
  - Ex. Group age 15-25, 26-80, and 80+
- Hinge functions: Allows us to fit a custom shape to the data. For example, we could have a slope from 15-25, then it would hinge at 25 until 80, again at 80 and above.
-

Part b: 0.5 point

Sample 1

- Binning: this adds variables and thus increase the degrees of freedom
- Piecewise linear functions: the hinge points need to be manually selected

Sample 2

- Does not account for variations within the bins
- Cut-off point selection is judgmental

Sample 3

- With binning variables, we may not have enough credibility in each of the buckets to have stable predictions
- For hinge function, these would have to be assigned by visual inspection

Sample 4

- For binning, this could lead to non-intuitive results, such as reversals.
- *(For piecewise)* Increases the degrees of freedom.

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
Candidates were expected to understand ways to address non-linearities within a GLM, and how to apply them to the example given, as well as its related downsides.
Part a
Candidates were expected to understand ways to address non-linearity within a GLM, and how to apply them based on the given plot. Common mistakes included: • Listing ways to address the non-linearity but not specifying how they can be applied to the specific example • Not explaining how the approaches can be applied to the given partial residual plot • Listing "adding interaction terms", "log transformations", or "adding offset terms" as the approach • For the hinge function, the candidate using an incorrect function form by including log transformation, for example, max (log(age) - 25, 0)
Part b

Candidates were expected to understand the downsides of the approaches in part a.
 	
Common mistakes included:
• Being too vague or too broad, for example:
    ○ "Binning will lose information."
    ○ For piecewise, "it is difficult to select break point."

QUESTION 6	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): A4**
SAMPLE ANSWERS	
Part a: 2 points	

Sample 1

F-test

$$F = \frac{D_S - D_B}{\Phi_S \cdot p_{added}} = \frac{1000 - 930}{1.75 \cdot 2} = 20 > 19.5 => reject\ H_O\ \&\ use\ Model\ B$$

D = $2(\ell\ell_{\text{sat}} - \ell\ell_{\text{model}})$

$D_S = 2(\text{-}1000 - (\text{-}1500)) = 1000$

$D_B = 2(\text{-}1000 - (\text{-}1465)) = 930$

=>second degree polynomial; adding 2 params

AIC = $-2\ell\ell + 2p$

AIC_S = -2(-1500) + $2p_S$ = 3000 + $2p_S$

AIC_B = -2(-1465) + $2(p_S + 2)$ = $\underline{2934 + 2p_S}$

66 > 0 -> model B better (smaller AIC)

Based on both F test & AIC, model B is better

=> Include age in rating plan

Sample 2

AIC = -2LL + 2p

AIC_A = -2(-1500) + 2p = 3000 +2p

AIC_B = -2(-1465) + 2(p+2) = 2930 +2p + 2

AIC_B = 2934 +2p < 3000 + 2p -> Model B is better based on AIC

BIC = -2LL + p•ln(n)

BIC_A = -2(-1500) + p•ln(n) = 3000 + p•ln(n)

BIC_B = -2(-1465) + (p+2)•ln(n)= 2930 + (p+2)•ln(n)

3000 + ~~p•ln(n)~~ = 2930 + ~~p•ln(n)~~ + 2•ln(n)

ln(n) = 35 n = e^{35}

Assume the number of observations (n) is less than e^{35}, so $BIC_B < BIC_A$ so Model B is better based on BIC as well

∴ I recommend including age since Model B performs better on both tests.

Sample 3

F-Test:

$$\text{F statistic } = \frac{2(\ell\ell_B - \ell\ell_A)}{\Delta parameter \cdot \Phi_{Small}}$$

$$= \frac{2(-1465+1500)}{2 \cdot 1.75} = 20 > 19.5$$

B is better than A

➔ Should include driver age

AIC: Assume # parameters in A is 20, since not given
AIC (A) = -2(-1500) + 2•20 = 3040
AIC (B) = -2(-1465) + 2•(20+2) = 2974
Since AIC is lower for B, B is superior

Thus I recommend using Model B as it performs better using the F test and AIC criteria.

Part b: 0.5 point

Sample 1
Because adding more variables into the model will always reduce the deviance statistic which will cause the model to be overfit.

Sample 2
Deviance isn't useful as adding more variables always decreases the deviance. Using AIC or BIC is more appropriate, as they penalize for adding new parameters.

EXAMINER'S REPORT

- Candidates were expected to know the formulas for F-test, AIC test, and/or BIC test and to be able to conclude whether the results from the test indicated inclusion or exclusion of the new variable.
- Candidates were expected to know that deviance and log-likelihood are inappropriate measures for comparing model structure in the situation given.
- Candidates were expected to be able to determine the number of parameters added to the model based on the given model form.

Part a

Candidates were expected to calculate two test statistics from any two of the F-test, AIC test, and BIC test, and to include a final conclusion that combined the results of both tests.

- Every test required that the candidate demonstrate that he/she knew how many parameters were added to the model.
- Only partial credit was given for using deviance or log-likelihood to compare models. These are not adequate for comparing nested models, which is the topic of part b. The conclusion could be made without calculating deviance as we know what it will be.
- Candidates should note that, as of this sitting, there is an ambiguity in the source text: the source paper does not adequately distinguish between deviance and scaled deviance. Therefore, the F-test statistic in this solution, while consistent with the source text, is technically incorrect. Using the correct methodology, the F-statistic would be calculated as follows:
  Model A Scaled Deviance = (2)(-1,000 + 1,500) = 1,000
  Model B Scaled Deviance = (2)(-1,000 + 1,465) = 930
  Model A Deviance = 1,000 * 1.75 = 1,750
  Model B Deviance = 930 * 1.75 = 1,627.5
  F stat = (1,750-1,627.5) / (2 * 1.75) = 35

Had a candidate performed the above calculation, they would have received full credit. The source text is currently being revised, and future candidates should make note of this when using this report as a study resource.

Common mistakes included:

- Failing to make an overall conclusion that combined the results of both tests
- Performing only one test
- Using 1 or 3 rather than 2 as the additional number of parameters
- Incorrect calculation of the deviance used in the numerator of the F-test. Specifically, candidates often forgot to multiply by two resulting in an F statistic that was half as large as it should have been and causing them to make the incorrect conclusion
- Assuming that the AIC or BIC could not be done without knowing the number of parameters. For AIC the number of parameters cancels out when comparing the models and becomes unimportant. For BIC, credit was given for either assuming a number of observations or stating at what cutoff the number of observations would change the conclusion. Credit was also given if candidates made a statement about the assumed number of parameters and/or the assumed number of observations.
- Using deviance in the AIC/BIC formula rather than log-likelihood or using log-likelihood rather than deviance in the calculation of the F-test.
- Making the wrong conclusion on a particular test even with the correct calculations.

Part b

Candidates were expected to know that deviance decreases or that log-likelihood increases with the addition of variables. Credit was not given if candidates simply said it improves.

Candidates were expected to know that using deviance alone would lead to over-fitting. Credit was also given to recognizing fitting to noise, as well as statements about penalizing for adding additional parameters.

Common mistakes included:

- Giving some of the limitations of deviance such as needing to have the same underlying dataset with the same distribution. This limitation is not restricted to deviance alone and it addresses situations where deviance or any test based on deviance is not appropriate at all for comparison rather than the question of why more than one test should be considered in the situations where deviance and tests based on deviance are appropriate to use for model comparison.

QUESTION 7	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): A3**
SAMPLE ANSWERS	
Part a: 0.5 point	
Policy 1: ln(0.013/(1-0.013) = -4.330 Policy 2: ln(0.203/(1-0.203) = -1.368 Policy 3: ln(0.025/(1-0.025) = -3.664	
Part b: 0.75 point	
Policy 1: ln(μ/(1-μ)) = -4.330+1.25-.02x78 = -4.640 μ = 1.0% Policy 2: ln(μ/(1-μ)) = -1.368+1.25-.02x92 = -1.958 μ = 12.4% Policy 3: ln(μ/(1-μ)) = -3.664+1.25-.02x35 = -3.114 μ = 4.3%	
Part c: 0.5 point	
Logit Function: ln(μ/(1-μ)) Value range (-∞,∞) Logistic Function: $1/(1+ e^{-\mu})$ Value range (0,1)	
Part d: 0.25 point	
Because the logistic function can take any value from (-∞,∞) and map it to a value between 0 and 1. This is also the range for probabilities, so it is an intuitive fit.	
Part e: 0.5 point	
Sample 1 Coverage-related options on a policy. This is because there are often counterintuitive relativities due to selection effects, so best to calculate in separate model and include as an offset. *Sample 2* Offset is useful for deductible, which is better estimated outside GLMs (e.g. LER analysis), since GLM often produces counterintuitive results due to effect of selection and correlation with variables outside model. *Sample 3* Territory rating is impractical to use in a GLM since there are hundreds or even thousands of territories with no easy way to group them without losing signal. However, territory differences are significant so it's important that the rating plan be offset for territory rates. Thus it's best to include territory factors as an offset in GLM. *Sample 4* If you're creating a model on renewal business after having already made a model for new business only, you would likely use an offset for many of the variables. This would ensure consistency between the sets of business that you do not expect to change over time.	

Sample 5
When including the effect of a coverage limit in a pure premium model. Limits may be correlated with other covariates not being accounted for in the model and this might lead to inconsistent ILFs based on model results, so it's better to do loss elimination analysis outside of the modeling process and include the effect of a coverage limit as an offset.

Sample 6
When introducing additional variables but do not want to change existing ones due to constraints like rate filing approval, IT system constraints, etc.

EXAMINER'S REPORT

Candidates were expected to know how to calculate an offset given the results of a prior model and then to use that offset in the new model with the addition of a new variable to calculate the new target variable (probability). They were also expected to know when it is appropriate to use an offset in a GLM. Candidates were expected to know the relationship between the logit function and logistic function and why the logistic function is appropriate for modeling probabilities.

Common mistakes included:
- Interchanging the logit and the logistic function
- Using the wrong link function
- Not tying all the pieces of the GLM (link function and linear predictor components such as intercept, insurance score and offset, etc) together

Part a

Candidates were expected to calculate the offset term to be used in the regression for each of the three policies.

Common mistakes included:
- Including the intercept or insurance score
- Using the incorrect link function

Part b

Candidates were expected to calculate the revised probability of a claim, using the model with the insurance score, assuming the old model as an offset.

Common mistakes included:
- Taking the natural log of the credit score
- Not including the insurance score (if it wasn't also erroneously included in part a)
- Not including the intercept (if it wasn't also erroneously included in part a)
- Not including the offset calculated in part a
- Not using the correct formula for μ

Part c

Candidates were expected to identify the range of the logit and logistic functions.

A common mistake was identifying the input range as opposed to the output range.

Part d
Candidates were expected to explain why logistic regression is used to model probabilities. Common mistakes included: • Just mentioning that it's in the probability range without giving what that range was • Mentioning that logistic regression uses a binomial response (Y/N) without tying it to the range of the logistic function • Mentioning the logit function/odds ratio, without tying it to the range of the logistic function
Part e
Candidates were expected to identify a situation in which an offset would be preferable as well as provide a description for that situation. A common mistake was not fully describing the situation or not fully providing an example.

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 8	
TOTAL POINT VALUE: 1.50	**LEARNING OBJECTIVE(S): B1**
SAMPLE ANSWERS	

Sample 1

Accuracy – the risk adjusted ILF is more accurate than the varying profit and contingency because it is more explicitly calculating the risk load as limits increase. The profit varying is more arbitrary.

Ease of Calculation – The risk load is much more computationally difficult. The profit can be more judgmentally selected.

Clarity – The risk load has a foundation in mathematics so would be more clear to a trained eye. But a lay person would likely better understand the profit variation.

Sample 2

1) The variance and standard deviation approaches are much more rigorous and do a better job of calculating risk loads from an accuracy perspective. They can calculate loads more precisely and wouldn't have to be bucketed like the proposed plan would.

2) The proposed plan would be easier to calculate risk loads for. A system of varying profit and contingency could be as simple as you'd like. On the other hand, the more complicated you make it, the closer you get to using risk-adjusted ILFs.

3) The risk-adjusted ILFs are clear in that the loading takes place behind the scenes (in the calculation of the ILF). The proposed method is presumably clear because the method of varying profit and contingency load would be explicitly laid out and simple to apply. In this sense, the proposed method is probably simpler.

Sample 3

Accuracy: Risk-adjusted ILF produces a more accurate premium. Varying the P/C load is a variable expense and will require us to take in a different amount of fixed expenses.

$\frac{p+f}{1-v}$ where f = fixed, v = variable, p = pure premium

This is less accurate because fixed expenses should not vary by limit. Risk-adjusted ILF allows us to calculate p directly and keep fixed expenses the same.

Ease of Calculation: Determining a risk load may require more work because the k constant will need to be calibrated to the portfolio. Profit and contingency needs to be set judgmentally for each limit, which may also take time but is far less technically rigorous.

Clarity: The varying P/C is likely more clear and transparent for those who don't have a thorough understanding of the portfolio. This is because it is explicitly defined in the premium calculation rather than buried in the pure premium calculation (which itself is an input into the premium calculation).

Sample 4

Accuracy: ILF derivation assumes that profit and contingencies are variable and the same across all limits. Adjusting the profit provision to vary with limit violates this assumption, therefore ILFs that have not been adjusted for risk load would be incorrect. Calculating risk load separately for each limit and using risk-adjusted ILFs is more accurate.

Ease of Calculation: Calculating separate risk loads for each possible limit is much more time consuming and calculation intensive compared to having a variable percentage applied to calculate the premium.

Clarity: It's unclear how the variable profit provision load would be determined using the alternate method, while the current method is a defined and reasonable approach.

EXAMINER'S REPORT

Candidates were expected to understand the calculation of ILF risk-adjustment as well as the purpose. This would then have been contrasted to a simpler profit/contingency approach across the categories of accuracy, ease of calculation, and clarity. For each of these three, candidates were expected to state what their position was on the better of the two and provide a reason for that opinion.

Candidates received no credit for:

- Stating a position across all categories without any attempt at stating a reason as to why
- Stating that both are the same method without detailing how that might be the case

Common mistakes included:

- Comparing risk-adjusted strictly to non-risk-adjusted without profit adjustment
- Focusing only on the calculation of the premium itself while not mentioning the work involved in determining risk-adjustment factors
- Stating the profit/contingency load would be harder to calculate because you already have risk loads
- Stating that risk adjustment is easier to calculate because there is a formula for it
- Not identifying or incorrectly identifying to whom one method versus the other may be clearer
- Assuming the profit/contingency load would be invisible and buried in rest of profit

QUESTION 9

TOTAL POINT VALUE: 4.25 | **LEARNING OBJECTIVE(S): B4**

SAMPLE ANSWERS

Part a: 2.75 points

Plan 1

Risk #	Manual	Loss ($)	Mod	Standard
2	$900	$490	0.68	$612
8	$1,300	$800	0.72	$936
10	$1,500	$975	0.76	$1,140
4	$975	$650	0.78	$761
5	$1,075	$850	0.88	$946
9	$1,450	$1,175	0.90	$1,305
6	$1,100	$1,000	0.96	$1,056
1	$810	$750	0.97	$786
7	$1,225	$1,300	1.06	$1,299
3	$950	$1,075	1.13	$1,074

Quintile	Manual Prem	Loss	Man LR	Std Premium	Std LR
1	$2,200	$1,290	58.6%	$1,548	83.3%
2	$2,475	$1,625	65.7%	$1,901	85.5%
3	$2,525	$2,025	80.2%	$2,251	90.0%
4	$1,910	$1,750	91.6%	$1,842	95.0%
5	$2,175	$2,375	109.2%	$2,373	100.1%

Variance Manual LR: 0.04110
Variance Standard LR: 0.00472
Efficiency Statistic= 0.00472/0.04110
= 0.1147

OR (Using population Variance):

Variance Manual LR: 0.03288
Variance Standard LR: 0.00378
Efficiency Statistic= 0.00378/0.03288
= 0.1147

Plan 2

Efficiency Statistic= 0.0059/0.0411
= 0.1436

Rank (Most to least Equitable): 3, 1, 2

Part b: 1.5 points

Sample Responses Insurer/Plan 1

- In insurer #1 standard LR's trend up, meaning there is not enough credibility given to experience. Safe risks charged too much and high risks not charged enough. Safe risks will leave and high risks stay – company will lose money. High risk of adverse selection.
- Plan 1 will attract risks with higher mods since the standard premium charged will be less than other competitors overtime the company will see an eroding LR and will become unprofitable.

Sample Responses Insurer/Plan 2

- In insurer #2 standard LR's trend down meaning too much credibility to experience. High risks charged too much and lower risks not charged enough. High risks will leave and low underpriced risks will stay. Company will lose money.
- Plan #2 will attract risks with lower experience mods because that premium charged will be lower than competition and overtime the company will see a shift to risks with lower mods but will still see an eroding LR.

Sample Responses Insurer/Plan 3

- Insurer #3 is ideal. Test statistic indicates standard LR's are flat meaning credibility to

experience correct and products being priced. Adverse selection risk is very low.
- Plan 3 does not favor any set of risks and will not create adverse selection as all risk groups are adequately adjusted.

EXAMINER'S REPORT

Candidates were expected to demonstrate the ability to assess experience rating plans, including calculating an efficiency statistic and describing how each plan performs.

Part a

Candidates were expected to calculate an efficiency statistic for Plan 1 and Plan 2, then rank the efficiency of all three plans.

Common mistakes included:
- Not grouping by quintiles
- Grouping the quintiles incorrectly
- Using standard deviation instead of variance in efficiency calculation

Part b

Candidates were expected to explain how adverse selection may impact each insurer assuming a well-functioning market.

Candidates who explained adverse selection but did not specify how this will impact each insurer directly received no credit.

Common mistakes included:
- Reversed which risks will be underpriced/overpriced between plan 1 and plan 2
- Not discussing the impact adverse selection will have on profitability
- Not discussing the impact of adverse selection on each of the insurers
- Omitting an explanation for insurer/plan 3

QUESTION 10	
TOTAL POINT VALUE: 2.25	**LEARNING OBJECTIVE(S): B3**
SAMPLE ANSWERS	
Part a: 1.25 points	

Sample 1

Not using 2018 period.
Assuming these are policy year LDFs.
Using Fisher method and calculating expected limited losses at current development and trend level.
Assuming expected ultimate loss at projected trend level is constant over time.

$$\text{Expected limited loss}$$
$$= 500K \times [1.5^{-1} \times 1.02^{-2} + 1.23^{-1} \times 1.02^{-3} + 1.2^{-1} \times 1.02^{-4}] \times \frac{17{,}500}{85{,}504}$$
$$= 222.758K$$

Sample 2

$$k = \frac{E(x) - E[x; 100{,}000]}{E(x)}$$

$$= \frac{85{,}504 - 17{,}500}{85{,}504}$$

$$= 0.7953$$

$$\% \text{ Limited} = 1 - 0.7953 = 0.2047$$

Year	Exp. Ult loss	% Lim.	Loss trend	LDF	Exp. Lim. loss
(1)	(2)	(3)	(4)	(5)	$(6) = \frac{(2) \times (3)}{(4) \times (5)}$
2017	500,000	0.2047	1.02^2	1.5	65,583.7498
2016	500,000	0.2047	1.02^3	1.23	78,411.9439
2015	500,000	0.2047	1.02^4	1.2	78,796.3161
					222,792.0098

$$\therefore \text{Expected reported losses } \approx 222{,}792$$

Part b: 1 point

Sample 1

$$\text{Hist capped loss} = 441{,}769$$

$$\text{Mod} = 1 + 0.6 \times \left(\frac{441{,}769}{222{,}758} - 1\right) = 1.590$$

Sample 2

Year (1)	Actual lim. loss (2)
2015	$3{,}450 + 5{,}000 + 234 + 98{,}000 + 100{,}000 = 206{,}684$
2016	$2{,}389 + 345 + 100{,}000 = 102{,}734$
2017	$456 + 100{,}000 + 2{,}345 + 1{,}874 + 690 + 26{,}986 = 132{,}351$
	$441{,}769$

$$\text{Mod Factor} = 1 + z\,\frac{A - E}{E}$$
$$= 1 + 0.6 \times \left(\frac{441{,}769 - 222{,}792}{222{,}792}\right)$$
$$= 1.59$$

EXAMINER'S REPORT

Candidates were expected to understand the actuarial principles and concepts underlying the development of experience rating plans. They were expected to demonstrate this knowledge by calculating the experience modification factor of a policy. This question was very similar to step 3 of the Exam 8 syllabus case study.

Part a

Candidates were expected to determine the appropriate trend period, to select the appropriate LEV from the table, and to apply the correct limited LDF in order to calculate the expected reported losses.

Common mistakes included:

- Using the wrong trend period
- Multiplying the LDFs as if they were age to age factors instead of factors to ultimate
- Using a trend of 4% instead of 2%
- Developing the historical losses to ultimate

Part b

Candidates were expected to apply the individual claim cap to reported losses, select the appropriate policy years, and calculate the experience modification factor using the credibility factor.

Common mistakes included:

- Using Policy Year 2018
- Using actual losses as expected losses, and vice versa

QUESTION 11	
TOTAL POINT VALUE: 1.25	**LEARNING OBJECTIVE(S): B3**
SAMPLE ANSWERS	
Part a: 0.5 point	

Sample 1
Mod = Z × (AER - EER) / EER

CSLC = 270,500
Z = 0.48
EER = 0.922

Mod = 0.48 × (0.85 – 0.922) / 0.922 = -0.0375
∴ Mod is a 3.75% credit

Sample 2
CSLC = 270,500
AER = 0.85

Look up CSLC, we get:
Cred = 0.48
EER = 0.922
MSL = 150,200

Mod = (AER - EER) / EER × cred
Mod = (0.85 – 0.922) / 0.922 × 0.48 = -0.0375

Sample 3
CSLC = 270,500
→ Z = 0.48
EER = 0.922
MSL = 150.2k

Mod = 1 + Z × (AER - EER) / EER
= 1 + 0.48 × (0.85 – 0.922) / 0.922
= 0.9625

Part b: 0.75 point

Sample 1
No credit for upgraded equipment because already reflected in experience.
Up to 6% for training.
Up to 2% for safety program.
10% is too high. The most they should get is 6% + 2% = 8%.

Sample 2
Experience period: 1/1/2016-12/31/2018

Upgraded equipment should be in the experience period → 0% credit.
Only one year of employee training in the experience period → 6% × (2/3) = 4%
Only one year is in the experience period for safety program → 2% × (2/3) = 1.3%
The schedule rating credit should be 0% + 4% + 1.3% = 5.3%.
Appears that the underwriter's selection is too much.

Sample 3
10% credit for schedule rating is excessive per ISO rating plan. Maximum credit mod for employee training is 6% and 2% for safety program. Also, the equipment upgrade in 2015 must be fully reflected in experience mod, thus no credit should be given. Therefore, I believe 8% credit schedule rating is more reasonable for this insured.

EXAMINER'S REPORT

Candidates were expected to be able to calculate an experience rating mod and determine a reasonable schedule rating mod given the information provided in the question and the ISO CGL Experience and Schedule Rating Plan.

Part a

Candidates were expected to look up the appropriate credibility and EER in the Credibility and Maximum Single Loss table using the Company Subject Loss Cost provided in the question. Candidates were then expected to calculate the experience modification using the ISO plan formula.

Common mistakes included:

- Neglecting to multiply by Z
- Using the wrong EER or CSLC

Part b

Candidates were expected to recognize that the equipment upgrade occurred prior to the start of the experience period, and therefore warranted no schedule credit. Candidates were also expected to state the maximum credits for employee training and safety programs found in the manual, and to conclude that the 10% credit is unreasonable given the maximum credits allowed.

Many candidates noted that the employee training and safety programs may be partially reflected in the experience (depending on when during 2018 they were implemented) and therefore do not deserve the maximum schedule credit. This was considered in awarding partial credit, but not necessary to receive full credit.

Candidates were expected to conclude that 10% was an unreasonable credit given that it exceeds the maximum credit of 8% allowed by the ISO manual. Credit was awarded to candidates who did not make a direct conclusion citing the 8% from the manual but instead recommended a reasonable smaller credit because the employee training and safety programs were already partially reflected in the experience period.

Candidates were not required to explicitly state that the experience rating period was 2016-2018.

A few candidates stated that they were assuming that the experience rating period was 2015-2017 because the full 2018 year was not yet available. These responses received full credit as long as they addressed all the required items noted above. A few others stated that the period was 2017-2019 (they were presumably ignoring the 6 month lag in the ISO manual). Candidates were able to reach the correct conclusion under this assumption.

The most common mistake was not looking up the allowable credits in the ISO manual and simply assuming either 5% or 10% for each item was reasonable without justification or reference to the ISO manual.

QUESTION 12	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE(S): B6**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1
Paid Loss Deductible = (Deductible Payments) + (Premium)
= 325,000 + [(450,000 – 325,000) + 450,000 × 0.8 + 100,000] × 1.03
= 593,830
Incurred Loss Retro
R = [100,000 + (450,000 – 325,000) × 1.08 + 325,000 × 1.08] × 1.03
= 603,580
593,830 < 603,580 so paid loss deductible is lower

Sample 2
Retro Plan
Expected insured claims = 450,000
E[R] = GCP = (450,000 × 1.08 + 100,000) × 1.03 = 603,580
Deductible Plan:
Exp ded payments = 325,000
Premium = [(100,000+ 0.08 × 450,000 + (450,000 – 325,000)] × 1.03
= 268,830
Total Cost = 268,830 + 325,000 = 593,830
Ded is 9,750 cheaper

Part b: 0.5 point

Sample responses recommending paid large deductible plan:
- The total cost (total premium + reimbursed losses) are lower (dependent on candidate's answer to part a)
- Lower total cost from not paying taxes on deductible payments
- Cash flow benefit from delayed payments for deductible losses versus up-front premium in the retro plan / the premium paid up front is lower providing a cash flow advantage

Sample responses recommending incurred retrospective rating plan:
- The total cost (total premium + reimbursed losses) are lower (dependent on candidate's answer to part a)
- Less credit risk so the insured has a lower cost of posting security/collateral
- Higher premium tax deductibility

Part c: 1 point

Sample responses assuming paid large deductible plan recommended in part b:
- Increase the aggregate deductible: this would lower premium while not significantly increasing risk to insured.
- Lower the aggregate deductible -> More certainty as less riskiness in the total cost for the whole PY.
- Increase the per-occurrence deductible. This would lower the premium + provide a bigger cash flow advantage to the insured.

- Lower the per-occurrence deductible. The insured would need to retain less of each loss, leading to less uncertainty.
- Change to incurred losses -> Pay more claims earlier, but less need to post collateral.
- Evaluate on an incurred basis -> Losses are more stable.
- Change from large dollar deductible to XS policy with self-insured retention. Insured can handle all claims below deductible and save on expenses built into insurer's policy while still getting XS loss protection.
- Structure as an SIR plan with same risk transfer structure -> Insured is charged less for LAE and has more control over claim adjusting. And the insurer may require less profit due to no credit risk and fewer services.
- Include LAE with loss and subject to deductible. This will reduce insured's loss share.

Sample responses assuming incurred retrospective plan recommended in part b:

- Propose a minimum ratable loss. The basic premium can be reduced.
- Adding a minimum premium or ratable loss would reduce swings in the retro premium.
- Lowering the max ratable loss would put less pressure on the insured if they had extremely high losses.
- Can opt for a retro development factor to limit premium swings.
- They could use a holdback which delays premium adjustments which will lower the back and forth between premium payments.
- Paid basis – If the insured wanted a cash flow advantage, they could use a paid loss basis instead of incurred loss basis.
- Use a multi-year plan to minimize rate fluctuations year over year.
- Multi-year plan to lock in low premium when market is soft.
- The retro plan can include a higher LCF (go above 1.08). This will give a cash flow advantage since insured will be paying less in basic premium.
- Have an additional dividend plan component where the insured will get dividend if the insurer's profit is over certain threshold.

EXAMINER'S REPORT

Candidates were expected to understand the pricing of loss sensitive rating plans, both retrospective rating and large deductibles. They were expected to compare the costs of the two plans with calculations, and to recommend one of the plans to the insured. Candidates were also expected to make suggested changes to their recommended plans and explain why they would be beneficial to the insured.

Part a

Candidates were expected to calculate the total cost of insurance for both the retro and deductible outlined, and compare the two. For the retro plan, this meant calculating the expected ultimate premium. For the deductible plan, this included calculating the upfront deductible premium and then adding the expected deductible reimbursements to the premium.

There were multiple ways to calculate the retro premium that received full credit. Candidates could:

- Calculate the basic premium and then use the LCF, ratable losses and tax multiplier to

calculate the expected ultimate premium
- Calculate the ultimate premium directly by adding up all of the components (UW expenses, excess losses, insurance charge, ratable losses, LAE on each of the loss components and tax)
- Calculate the expected retro premium assuming it was equivalent to a guaranteed cost plan

Common mistakes included:
- Treating the 100,000 given in the problem for "Fixed underwriting expenses including commission and profit" as 'e', the expenses underlying a guaranteed cost plan, which includes LAE(LAE was given separately in the question)
- Neglecting to accurately identify the expected excess losses and/or insurance charge to be included in the basic premium / deductible premium
- Neglecting to apply the LAE % to correct loss amount, or not including LAE at all
- Calculating only the deductible premium and not adding the deductible reimbursements, or the incorrect amount of deductible reimbursements
- Applying the tax multiplier and/or LAE % to the reimbursements for the deductible plan
- Neglecting to compare the total cost of the two plans
- Not showing any calculations but attempting to compare how the two plans differ qualitatively

Part b

Candidates were expected to be able to make a recommendation of which plan was preferred and provide one item of support. An explicit plan recommendation was required for full credit.

Candidates had the option of referring to plan total costs calculated in part a for their justification, discussing the general characteristics of a retrospective rating plan versus large deductible plan (or vice versa), or some combination of both.

Common errors included:
- Giving support for the plan recommendation that was either relevant to a discussion of a guaranteed cost versus loss-sensitive plan
- Providing a description that was too general or vague
- Providing a benefit for the insurer rather than the insured.

Part c

Candidates were expected to list and justify two changes to the insurance plan they recommended in part b that are beneficial for the insured.

If candidates failed to recommend any plan in part b, credit was awarded provided they clearly stated which plan the changes were for.

Common mistakes included:
- Proposing changes that are benefits to the insurer rather than the insured
- Proposing changes that don't apply to the plan recommended in part b
- Proposing to change the underwriting expenses including profit and commission to either be

variable or negotiating a lower fixed dollar load

- Listing only one change
- Lack of or insufficient support for why the suggested change is beneficial for the insured
- Confusion in the directions of premium changes when deductible (either per-occurrence or aggregate) changes in an LDD plan
- Stating that changing the minimum/maximum ratable losses would lead to a change in ultimate premium in a retro plan

QUESTION 13	
TOTAL POINT VALUE: 2.75	**LEARNING OBJECTIVE(S): B1, B2**
SAMPLE ANSWERS	
Part a: 0.75 point	

Sample 1

P(X>10,000) = 1 – F(10,000)

$= \left(\frac{\beta}{\beta+10{,}000}\right)^{\alpha} = \left(\frac{6{,}000}{6{,}000+10{,}000}\right)^{2} = 0.140625$

E[N >10,000] = E[N] × P(X>10,000) = 5,000 (0.140625) = 703.125

Sample 2

Since Poisson, can use E[N] × [1- F(10,000)]

E[N] = λ = 5,000

$F(10{,}000) = 1 - \left(\frac{6{,}000}{16{,}000}\right)^{2} = 55/64$

1- F(10,000) = 9/64

=> Expect 9/54 × 5,000 = 703.125 excess claims

Part b: 2 points

Sample 1

Change in agg losses:

$$\frac{1.03\times\left(E[X]-E[X;{}^{10000}/_{1.03}]\right)}{E[X]-E[X;10000]} = \frac{1.03\times(6000-3708)}{6000-3750} = 1.0491$$

$E[X] = \frac{\beta}{\alpha-1} = 6000$

$E[X;\ 10000] = \frac{6000}{2-1}\times\left(1-\left(\frac{6000}{16000}\right)^{2-1}\right) = 3750$

$E[X;\ {}^{10000}/_{1.03}] = 3708$

For $\tilde{\tau}_S$ = 1.00 (no change in agg), must have claim count impact

$\tilde{\tau}_N$ = 1 / 1.0491 = 0.9532

So % change in ground up counts = 0.9532 – 1 = -4.68%

Sample 2

E[S] = E[N] × $(E[X]-E[X;10{,}000])$ α=2 ; $\beta = 6{,}000$

$= 5000\times\left(\frac{\beta}{\alpha-1}-\frac{\beta}{\alpha-1}\left[1-\left(\frac{\beta}{10000+\beta}\right)^{\alpha-1}\right]\right)$

$= 5000\times\left(\frac{6000}{2-1}-\frac{6000}{2-1}\left[1-\left(\frac{6000}{16000}\right)^{2-1}\right]\right)$

= 50000 × 60000 × 6/16 = 11,250,000

E[S'] = E[N'] × $(E[Y]-E[Y;10{,}000])$ Y = 1.03X

= E[N'] × 1.03 × $(E[X]-E[X;10{,}000/1.03])$

= 1.03 × E[N'] × $\left(6000-6000\left[1-\frac{6000}{60000+9708}\right]\right)$

= 1.03 × 6000 × $\frac{6000}{15708}$ × E[N'] = 11,250,000

E[N'] = 4766 => rate = $\frac{4766-5000}{5000}$ = -4.68%

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
Candidates were expected to know how to calculate the expected number of excess claims, how to calculate expected loss, how to calculate excess and/or limited loss, and how inflation impacts loss calculations. The candidate was then expected to know how to derive the aggregate impact of inflation on excess loss and how this is impacted by a change in ground-up claim costs.
Part a
The candidate was expected to know how to use the parameters and formulas given on the Poisson claim count distribution and Pareto claim size distribution in order to calculate the expected number of claims in the excess layer. Common mistakes included: • Calculating the number of claims not hitting the excess layer • Calculating the probability of an excess claim, but stopping before calculating the number of claims in the excess layer • Using $\alpha - 1$ in the probability formula instead of α • Using F(x) instead of 1-F(x) to get the expected number of claims in the excess layer
Part b
Candidates were expected to know how to take claim count and claim size distributions and use them to determine the rate of change necessary in ground-up claims counts to offset the impact of inflation in a specified layer of aggregate excess losses. Most successful candidates took one of two paths to calculate the rate of change needed. The first set of candidates calculated the expected loss, limited expected loss, and modified expected loss need in the $\tilde{\tau}_S$ calculation, and then calculated the rate of change needed as $1/\tilde{\tau}_S - 1$. The second set of candidates calculated the expected total aggregate excess loss before inflation and then calculated the expected ground-up claims needed in order for that aggregate figure to stay the same after inflation. The candidates then derived the rate of change as the new ground-up claims divided by the old ground-up claims minus one. Common mistakes included: • Calculating the aggregate change as 1.0491 and saying the rate which ground-up claims needed to change was (1.0491-1) = 4.91% • Transposing the numerator and denominator in the excess loss ratio comparison • Using limited loss in the ratio comparison instead of excess loss • Stating a new potential ground-up claim number and not the rate ground-up claims needed to change • Misapplying the impact of inflation on excess loss occurrence probability in the calculation, when trying to calculate the total change to the aggregate excess layer

QUESTION 14	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE(S): B5**
SAMPLE ANSWERS	
Part a: 1.25 points	
Sample 1 R = [B + c(E – I)] × T E = 4,860,000 / 0.9 = 5,400,000 Charge (R_g)= 0.0379 (lku from table) Charge (R_h) = 0.7253 (lku from table) Savings (R_h) = 0.0053 I = (Charge – Savings) × E = (0.0379 – 0.0053) × 5,400,000 = 176,040 B = 2,250,000 B/SP = 2,250,000 / 9,000,000 = 25% *Sample 2* B/SP = e/SP – (c-1) ×E / SP + cI/SP GCP = 9,493,205 = (e + E) × T e = 3,641,148 Charge (R_g)= 0.0379 (lku from table) Charge (R_h) = 0.7253 (lku from table) Savings (R_h) = 0.0053 I = (Charge – Savings) × E = (0.0379 – 0.0053) × 5,400,000 = 176,040 B/SP = 2,250,000 / 9,000,000 = 25%	
Part b: 1.25 points	
Sample 1 I(current) = 176,040 from part a Charge(R_g) = (1.65 – 1.55)/(5×0.6) = 0.04 Savings(R_h) = (0.168 – 0.15)/(5×0.6) = 0.006 I (experience) = 0.6 × (9,000,000) × (0.04 – 0.006) = 183,600 The net insurance charge from a is lower than calculated from the prior experience *Sample 2* I(current) = 176,040 / 9,000,000 = 0.0196 from part a Charge(R_g) = (1.65 – 1.55)/(5×0.6) = 0.04 Savings(R_h) = (0.168 – 0.15)/(5×0.6) = 0.006 I (experience) = 0.6 × (0.04 – 0.006) = 0.0204 The net insurance charges are fairly comparable *Sample 3* I(current) = 0.0326 from part a Charge(2.55) = (2.75 – 2.55)/(5) = 0.04 Savings(0.28) = (0.28 – 0.25)/(5) = 0.006 I (experience) =0.04 – 0.006 = 0.034 The insurance charge is slightly higher using the experience	

Part c: 0.5 point

Sample 1

Basic premium should not be derived using prior loss experience because prior carrier could have had completely different expense loads.

Sample 2

If the max or min entry ratios have changed, it is not appropriate to use the prior carrier experience to calculate basic premium.

Sample 3

Not appropriate because it's only 5 years of data, which is not very credible.

Sample 4

In this case I don't think it is appropriate. There is not enough data and the loss ratios are very volatile.

Sample 5

The prior carrier could have a different mix of business than the current carrier so aggregate distributions may not be the same.

EXAMINER'S REPORT

Candidates were expected to be able to calculate a basic premium and insurance charges using values looked up in the NCCI Retrospective Rating manual as well as using the prior loss experience. They were also expected to opine on whether it is appropriate to use prior loss experience in determining basic premium.

Part a

Candidates were expected to calculate basic premium as a percentage of standard premium using the given information and the NCCI Retrospective Rating manual. Some candidates calculated the expense ratio to determine the basic premium as shown in Sample Response 2, which was a valid response but not necessary for full credit.

Common mistakes included:

- Not calculating the insurance charge
- Not subtracting the insurance charge from the total expected loss
- Not using the Adjusted Expected Loss to determine the correct charge and savings from the NCCI Retrospective Rating manual and instead deriving some other expected loss
- Not dividing the final calculated basic premium by the standard premium
- Using the expense ratio from the NCCI Retrospective Rating manual and not deriving the expense ratio from the given information (if using the method in Sample 2)

Part b

Candidates were expected to calculate the net insurance charge based on the insured's prior loss experience and compare to the net insurance charge from part a. Candidates received credit whether this comparison was done comparing the net insurance charge as a dollar amount, as a %

of Standard Premium, or as a % or Expected Loss. Some candidates constructed a Table M or Lee diagram to determine the insurance charge and savings from the prior loss experience, which was an unnecessarily complicated but valid approach. • The most common mistake was making the comparison on a different basis (i.e. % Expected Loss vs % Standard Premium).
Part c
Candidates were expected to discuss the appropriateness of using an insured's prior loss experience from another carrier to derive a basic premium charge. Common mistakes included: • Not stating a position on the appropriateness • Not supporting the given position

QUESTION 15	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): C4**
SAMPLE ANSWERS	

Sample 1

No loss corridor the expected LR is
$37.3\times0.03 + 53.2\times 0.21 + 66.1\times0.55 + 91.1\times0.21 = 67.777$

With loss corridor, the expected LR is
$0.9\text{-}0.2\text{-}0.05\text{-}0.01\text{-}0.01 = 0.63$

So overall the loss corridor reassumes 67.777-63 = 4.777% of losses.

$37.3 \times 0.03 + 53.2 \times 0.21 + (60 + 6.1x) \times 0.55 + (60 + 20x + 11.1) \times 0.21 = 0.63$
$x = 0.368$
So the loss corridor reassumes $1 – 0.368 = 0.632$.

Sample 2

Expected loss ratio – 90%-20%-5%-1%-1% = 63%
Expected primary insurer loss ratio = $37.3\times0.03 + 53.2\times0.21 + 66.1\times0.55 + 91.1\times0.21 = 67.777$

x = insurance company reassumed

$0.373\times0.03 + 0.532\times0.21 + [0.60+(1\text{-}x)(0661\text{-}0.6x)]\times0.55 + [0.60+(1\text{-}x)(0.8\text{-}0.6)+(0.911\text{-}0.8)]\times0.21 =$
0.63
x = 63.2%

Sample 3

$ELR_{WO} = 37.3\times0.03 + 53.2\times0.21 + 66.1\times0.55 + 91.1\times0.21 = 67.777$

$ELR_{with}=0.03\times0.373 + 0.21\times0.532 + 0.55(0.661\text{-}0.061x) + 0.21(0.911\text{-}0.2x) = 0.67777 – 0.075555x$
$= 90\%\text{- } 5\%\text{-}20\%\text{-}1\%\text{-}1\% = 0.63$
x = 63.23% is reassumed by the primary insurer.

Sample 4

Expected Combined ratio = expected loss ratio + expected commission + expense ratio = 90%
Expected loss ratio = 63%
Expected loss ratio minus assumed = $0.03\times37.3\% + 0.21\times53.2\% + 0.55\times(66.1\text{-}6.1x) + 0.21(91.1\text{-}20x)$
$63\% = 67.777 – 7.555x$
$4.777 = 7.555x$
x = 63.2%

Sample 5

Primary reassumes x% of corridor losses

Range	Reassumed	P	E[reassumed]	Avg LR	E[LR]
1	0	0.03	0	37.3	1.1
2	0	0.21	0	53.2	11.2
3	6.1%x	0.55	0.03355x	66.1	36.4
4	0.2x	0.21	0.042x	91.1	19.1
			0.7555x		67.8

Combined Ratio = 90% = Avg LR – Reassumed + Expense = 67.8 – Reassumed +27
Reassumed = 4.8%

0.48 = 0.07555x
x = 63.5%

Sample 6

90% - 20% - 5% - 1% - 1% = 63%

(66.1%-60%)×0.55 + 20% × 0.21 = 7.56%
37.3×0.03 + 53.2× 0.21 + 66.1×0.55 + 91.1×0.21 = 67.78%
67.78% - 7.58% × X = 63%
X = 63.2%

Sample 7
Ceded ELR = 90-20-5-1-1 = 63%
E Tot LR = 37.3×0.03 + 53.2×0.21 + 66.1×0.55 + 91.1×0.21 = 67.78%
E assumed LR = 67.777% - 63% = 4.777%
(66.1-60)×.55 +20%×.21 = 7.55%
% assumed = 4.777%/7.555% = 63%

EXAMINER'S REPORT

Candidates were expected to be able to determine the effect of a loss corridor on the price of a reinsurance contract. The candidate was expected to calculate the target loss ratio including the loss corridor. At that point, the candidate could proceed in a few ways: either calculating the primary insurer expected loss ratio, calculating the loss in the corridor, and getting the needed loss ratio reduction, or calculating the loss percentage kept by the reinsurer and subtracting from 1.

Common mistakes included:

- Calculating the percentage kept by the reinsurer, and then not converting that to the percentage reassumed by the primary
- Calculating the expected loss ratio (including loss corridor) incorrectly, by assuming that some of the expenses were allocated by premium

QUESTION 16	
TOTAL POINT VALUE: 1.75	LEARNING OBJECTIVE(S): C1, C2
SAMPLE ANSWERS	

Part a: 1 point

Sample 1

Return Period (RP)	Exceedance Probability (EP)	Gross PML	Ceded PML	Net PML
1000	0.001	125	50	75
500	0.002	105	50	55
200	0.005	95	45	50
100	0.01	70	20	50
50	0.02	50	0	50

Interpolate Gross PML for 250 between (.002, 105) and (.005, 95)

95 + (.005-.004)/(.005-.002) = 98.33M

Ceded PML = 98.33 - 50 = 48.33

Net PML = 50 M

Sample 2

Gross PML = (.004 - .002) / (.005 - .002) × 95 + (.005 - .004) / (.005 - .002) × 105 = 98.3M

Ceded PML = 2/3 × 45 + 1/3 × 50 = 46.6m

Net PML = 2/3 × 50 + 1/3 × 55 = 51.6m

Sample 3

Assume contract is occurrence XOL and hurricane largest event is larger than EQ.

Therefore, no EQ event large enough to hit layer & ceded PML is 0.

Net PML is equal to gross: 1/500 = .002, 1/200 = .005

Linearly interpolate: (105) – (0.004 – 0.002) / (0.005 – 0.002) × 10 = 98.3

Part b: 0.75 point

Sample 1

Reinsurer new PML = 825 + 48.33 = 878.33

The PML 873.33 is the max number. PML cannot be added.

We need to re-run model to see the marginal impact 873.33 is greater than capital of 850M.

However, if we re-run model and incorporate the diversification benefit from EQ exposure due to this new contract, the PML may be reduced to an acceptable level. Therefore, the reinsurer should accept.

Sample 2
The reinsurer's 1-in-250 PML is driven completely by hurricane, so it should participate in the treaty to diversify the perils it is exposed to. Since the largest hurricane event for the primary insurer is 45m, the reinsurer will not increase its exposure to hurricane – only earthquake. This means the reinsurer's 1:250 should not grow by taking on additional EQ exposure. This is all, of course, on a modeling basis. There is potential for the model to be wrong, but the reinsurer should diversify.

Sample 3
Maximum Hurricane loss is 45M which is below retention and would trigger no payment from reinsurer. PML for hurricane would not increase (stay at 825M) for reinsurer. PML for EQ would increase by ceded PML. Total aggregated PML = Sq Root($46.67^2 + 825^2$) = 826.32 < 850.
Yes, Reinsurer should participate.

Sample 4
The reinsurer has 850-825 = 25 M of available capital. The 48.3 M PLM could seem too high, but it depends how this possible loss is correlated with the current book of business.

Since the current book is solely driven by hurricane peril and that this proposed contract is driven by both hurricane peril and EQ peril,

I would recommend to accept. Furthermore, the insurer never registered a hurricane >45M which is under the attachment point of the treaty.

Sample 5
Assume contract is occurrence XOL and hurricane largest event is larger than EQ.
Therefore, no EQ event large enough to hit layer & ceded PML is 0. The reinsurer should participate in the treaty because there are no chances of loss hitting the layer. Typically, reinsurers will write cat insurance if Pr(L > Prem + Surplus) < p where p is desired insolvency threshold & L is the PML.

EXAMINER'S REPORT

Candidates were expected to be able to apply reinsurance terms using an exceedance probability curve, as well as understand the effect of catastrophe risk in portfolio management.

Due to the ambiguity in outlining the type of reinsurance contract being offered and the primary insurer's PML table incorrectly being labeled "aggregate," this question was ruled to be defective. This defect was addressed through the grading of the question, as discussed herein. The intent of the question was for the PML table to be interpreted as occurrence. Most candidates answered the question as intended, as if it were an OEP.

Part a

Candidates were expected to interpolate the exceedance probabilities and then correctly apply the reinsurance contract terms to calculate the primary insurer's 1-in-250 ceded and net PML.

Candidates who misinterpreted the PML table and attempted to create their own OEP table received partial credit if the reinsurance terms were applied correctly. Also, candidates who did not interpolate using exceedance probabilities were awarded partial credit if the reinsurance terms were applied correctly.

Candidates who calculated AALs and applied reinsurance terms to these totals received no credit, as the question asks for the 1:250 PML.

The fact that the PML table was labeled "aggregate" did not seem to cause many candidates issues. Many candidates performed the calculations as if the table was on an occurrence basis, as the question intended. The candidates who recognized the issue and stated their assumptions were graded as if their assumptions held. Candidates who stated a valid reason for not interpolating, and then applied the reinsurance terms correctly were given full credit.

Common mistakes included:

- Using return period years for interpolation
- Switching the weights to be applied in the interpolation calculation
- Switching the ceded/net losses for the primary insurer.

Part b

Candidates were expected to recognize the diversification benefit to the reinsurer by writing the treaty, the effect this has on the reinsurer's capital, and then recommend taking on the risk.

Candidates who recognized the error in the question and provided sound arguments were given full credit for this part.

Generic arguments about reinsurance or answers given from the primary insurer's perspective were not given credit, as it did not address the information given in the question. For example, "if the premium charged is adequate, the reinsurer should write the contract," would receive no credit. Only arguing about potential "free cover" was also not given credit.

Some candidates incorrectly viewed this question from an experience/exposure rating perspective. This was not accepted, as the question deals with catastrophe modeled losses.

Part b becomes unanswerable with the assumption that the treaty is per-occurrence, combined with an aggregate PML table as we do not have enough information. Candidates who recognized this issue and explained why received full credit.

Common mistakes included:

- Ignoring the diversification effect
- Directly adding the primary insurer's 1:250 PML to the reinsurer's current PML.

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 17	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): C2**
SAMPLE ANSWERS	
Part a: 0.5 point	
Sample 1 Vulnerability module quantifies the physical damage on properties at risk Inventory module describes properties at risk *Sample 2* It will impact both inventory module and vulnerability module: Because the insurer needs to update properties' information regarding this new construction technique in the inventory module and incorporate its susceptibility to loss damage in the vulnerability module. *Sample 3* This change would primarily affect the vulnerability module which relates to how susceptible different building types are to damage from a catastrophe. The change would require the use of engineering judgment, building response analysts, or class-based building response analysis to create revised assumptions. The change would also require an update to the inventory module to make sure that buildings of the new type in the insurer's portfolio are appropriately identified as such.	
Part b: 0.5 point	
Sample 1 Epistemic, because it's due to lack of data. *Sample 2* It will be epistemic. We may not know enough about this technique, and there's no historical data for us to model it, causing uncertainty. *Sample 3* This is epistemic uncertainty since it is parameter risk rather than process risk inherent to the nature of the catastrophe. The epistemic uncertainty could be mitigated with greater scientific knowledge. *Sample 4* Aleatory is inherent randomness from natural hazard events (CAT version of process risk); epistemic risk stems from lack of knowledge about a hazard. This new construction technique creates epistemic uncertainty since we have no historical data on its performance and little scientific data to base our estimates on.	
Part c: 1 point	
Sample 1 • Logic Tree assigns weight to parameter alternatives based on expert opinion. A weighted linear combination is calculated. This relies on simplified opinions but is easy to communicate. • Simulation creates randomly sampled alternatives from a probability distribution of the parameter. It can handle complex situations but is difficult to compute.	

Sample 2

- Incorporate uncertainty by assigning different weights (probabilities) to parameters using a logic tree. This addresses epistemic uncertainty. This method maybe somewhat difficult to update and requires reliance on expert opinion. However, it is potentially easier to understand.
- Use simulation techniques to attempt to recreate the inherent randomness of the catastrophe. This addresses aleatory uncertainty. Simulation techniques can incorporate more robust assumptions and data and can be updated more easily. However, these models require more calculation and can be difficult to understand.

Sample 3

- Logic Trees can be used to incorporate uncertainty. Probability can be assigned to various parameters (magnitude, soil type, location, wind speed, etc.) and probabilities and parameter values are multiplied together to get expected loss. This method is easy to trace and understand, but simple and not easy to scale to a large number of scenarios.
- Simulation can also be used to account for uncertainty in modeling. These are complex scenarios that are run thousands of times based on probabilities of various parameters to estimate expected loss. These methods are computationally complex and may be a "black box" to those who don't understand the mechanics.

Sample 4

- Add a risk load to the expected loss when calculating the premium. Risk load could be some percentage of standard deviation of expected annual aggregate losses. This may be easier to communicate and calculate, but it is judgmentally selected and more difficult to justify.
- Credibility weight with and without effects of the new science to get credibility weighted damage function, with the compliment of credibility being no inclusion of non-consensus science. This may be more stable, as it will have less major change year over year until new science becomes more mainstream and generally accepted.

Sample 5

- Run multiple models with different vulnerability assumptions for the new technique. Compare the models to gauge impact. This is "sensitivity testing" to get an idea of how much the change impacts the overall model.
- Add a probability distribution around the vulnerability assumptions with the model. This will incorporate uncertainty directly into the model.
- The first method may be more reliant on expert opinion, which are judgmental and could be biased.
- The second method gives a better overall view of the uncertainty in future results but is more computationally intensive.

EXAMINER'S REPORT

Candidates were expected to describe the components and structure of catastrophe models, understand the sources of uncertainty in modeling, and illustrate the basic mechanics of uncertainty in models.

SAMPLE ANSWERS AND EXAMINER'S REPORT

Part a
Candidates were expected to identify the inventory and vulnerability modules and provide a brief description of each. A common mistake was to only describe one module.
Part b
Candidates were expected to identify the uncertainty as epistemic and briefly justify why. Common mistakes included: • Conflating epistemic risk with aleatory risk • Providing an explanation without selecting the type of uncertainty
Part c
Candidates were expected to name and describe two distinct methods of incorporating uncertainty and list two ways in which the given methods materially differ. Candidates most commonly used logic trees and simulation as methods to incorporate uncertainty. Alternative answers earned credit if they described a way to quantify and integrate *multiple* parameter estimates, *multiple* model outputs, *or* a specific way to add a risk load to the expected losses. Common mistakes included: • Providing no description or only a vague description (e.g. "add a risk load", "increase the variance") • Neglecting to include contrasting qualities of the two methods • Simply describing a component of the vulnerability module